Algebra I, Algebra II, Geometry, & 21st Century Math Applications Curriculum

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High School Mathematics Curriculum Work Committee

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Curriculum Formatting and Compilation

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Introduction

This document serves to meet all requirements for curriculum as per the Middle Township Board of Education and the New Jersey Department of Education and will serve as a guide for lesson planning. Units within the curricular framework for mathematics are designed to be taught in the order in which they are presented. There is a logical and developmentally-appropriate progression of standards, with strong consideration given to Major, Supporting, and Additional content standards presented since most concepts build upon each other. Within the units, the teachers have flexibility of what order to present the standards. Major, Supporting and Additional clusters of mathematics content standards are based on the New Jersey Student Learning Standards. Suggested Mathematical Practice Standards are listed in each unit to be imbedded regularly in daily math instruction.

Course Description

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the

coordinate plane. Two or more equations and/or inequalities form a system. A solution f or such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number syst em, but have a solution in a larger system. For example, the solution of *x+* 1= 0 is an integer, not a whole number; the solution of 2x + 1= 0 is a rational number, not an integer; the solutions of $x^2 — 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.
The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b2)/2)h,$ can be solved for *h* using the same de Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for findi ng approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathe matics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropri ately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). The eight standards for mathematical practices are as follows:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary,

middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards, which set an expectation of understanding, are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Pacing Guide

UNIT TITLE	ENDURING UNDERSTANDINGS	NJSLS	TIMEFRAME
Problem Solving in All Units	Understanding of what has occurred, why the process was effective, and can justify this with sound reasoning and evidence. Multiple strategies may be used to solve problems, although some are more effective than others. Reasonableness of solutions is determined by the context of the problems.	***Standards for Mathematical Practice MP1 through MP8***	Integrated Throughout Units/Course
1 - Solving Linear Equations and Inequalities	**College Prep:** 1. Solve equations as a process of reasoning and will be able to explain the reasoning 2. Solve equations and inequalities in one variable 3. Represent and solve equations and inequalities graphically 4. Create equations that describe numbers or relationships 5. Interpret the structure of expressions Advanced: The above CP understandings plus the following: 7. Reason quantitatively and use units to solve problems 8. Use properties of rational and irrational numbers	***All Levels:*** **A.REI.A.1** **A.REI.B.3** **A.CED.A.1** **A.CED.A.4** ***Advanced Only:*** **N.Q.A.1** **N-RN.B.3**	CP: 58-60 days Adv: 50-52 days
2 - Introduction to Functions, Linear Functions, & Exponents and Exponential Functions	CP: 1. Interpret functions that arise in applications in terms of the context 2. Analyze functions using different representations 3. Understand the concept of a function and use function notation 4. Represent and solve equations and inequalities graphically 5. Construct and compare linear and exponential models and solve 6. Create equations that describe numbers or relationships	***All Levels:*** **F-IF.A.1 & 2 & 3** **F-IF.B.4 & 5 & 6** **S-ID.C.7** **N-RN.A.1** **A-CED.A.2 & 3**	CP: 58-60 days Adv: 50-52 days

	7. Extend the properties of exponents to rational exponents Adv: The above CP understandings plus the following: 8. Reason Quantitatively and use units to solve problems 9. Use coordinates to prove simple geometric theorems algebraically 10. Summarize, represent, and interpret data on two categorical and quantitative variables 11. Build new functions from existing functions 12. Build a function that models a relationship between two quantities	***Advanced Only:*** **N-Q.A.3** **S-ID.B.6a & b & c** **S-ID.C.8 & 9** **F-BF.A.1 & 2** **F-BF.B.3** **N-RN.A.2** **F-IF.C.7a & b** **F-IF.C.9** **F-LE.A.1** **F-LE.A.1a & c** **F-LE.A.2** **F-LE.B.5** **A-SSE.A.1b** **A-SSE.B.3c**	
3 - Systems of Equations, Polynomials & Factoring, Quadratic Functions & Equations, and Rational Expressions & Functions	CP: 1. Representing and solving equations and inequalities graphically 2. Solving equations and inequalities in one variable 3. Performing arithmetic operations on polynomials 4. Interpreting the structure of expressions 5. Creating equations that describe numbers or relationships 6. Interpreting functions that arise in applications in terms of the context Adv: The above CP understandings plus the following: 7. Understand the concept of a function and use function notation 8. Analyzing functions using different representations 9. Solving systems of equations	***All Levels:*** **A-REI.B.4a & b** **A-REI.D.11 & 12** **A-APR.A.1** **A.SSE.A.1** **A.SSE.A.1a** **A.SSE.A.2** **A.SSE.B.3a & b** **A-CED.A.1** ***Advanced Only:*** **A-REI.C.5, 6 & 7** **F-IF.A.2** **F-IF.B.4 & 6**	CP: 58-60 days Adv.: 49-51 days

		F-IF.C.7 & 8 & 9 **F-IF.C.7a** **F-IF.C.8a** **F-BF.A.1** **F-BF.B.3** **A-CED.A.2 & 3** **S-ID.B.6a & b** **F-LE.A.3** **A-APR.B.3** **A-SSE.B.3**	
4 - Radical Expressions and Equations plus Data Analysis and Probability	1. Simply radicals and radical expressions 2. Graph square root functions 3. Use a variety of ways to organize and represent Data(Ex: Matrix, histogram, Box-&-Whisker plot) 4. Permutations & Combinations 5. The difference between Theoretical and Experimental Probability	***Advanced Only:*** **F-IF.B.4 & 5 & 6** **F-IF.C.7b** **F-BF.A.1b** **F-BF.B.3** **F-BF.B.4** **F-BF.B.4a** **S-ID.A.1 & 2 & 3** **S-ID.B.5**	Adv.: 23-25 days

Unit Plans

Content Area:	Algebra 1	Grade(s): 9-12
Unit Plan Title:	Unit 1: Solving Linear Equations and Inequalities	

Domain(s)

College Prep (CP): A.REI(Reasoning with Equations & Inequalities), A.CED(Creating Equations), & A.SSE(Seeing Structure in Expressions)

Advance(ADV): all CP standards plus N.Q (Quantities) & N-RN(The Real Number System)

Overview/Rationale

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

Equations & Inequalities: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Numbers & Number Systems: In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by imaginary numbers to form complex numbers.

Quantities: Quantities has a real application to real world problems. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Content Standard(s) (Established Goals)

College Prep:
A.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equations has a solution. Construct a viable argument to justify a solution method.
A.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V=IR to highlight resistance R.*
A.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic function, and simple rational and exponential functions.*

Advanced: All College Prep standards plus:
N-Q A.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N-RN.B.3: Explain why the sum or product numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments & critique the reasoning of others.
MP.4 Model with mathematics.
MP.5 Use appropriate tools strategically.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeating reasoning.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.
8.1.5.A.3: Use graphic organizers to organize information about problem or issue.
8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.
8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or

models.
8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.
8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.
8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.
8.2.2.E.1: List and demonstrate the steps to an everyday task.

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

College Prep:
1. Solve equations as a process of reasoning and will be able to explain the reasoning
2. Solve equations and inequalities in one variable
3. Represent and solve equations and inequalities graphically
4. Create equations that describe numbers or relationships
5. Interpret the structure of expressions

Advanced: The above CP understandings plus the following:
7. Reason quantitatively and use units to solve problems
8. Use properties of rational and irrational numbers

Essential Question(s) : (What provocative questions will foster inquiry, understanding, and transfer of learning?)

1. How can you represent quantities, patterns, and relationships?
2. How are properties related to algebra?
3. Can equations that appear to different be equivalent? Explain your reasoning.
4. How can you solve equations? Explain the process.
5. What kinds of relationships can proportions represent?
6. How do you represent relationships between quantities that are not equal?
7. Can inequalities that appear to be different be equivalent? Explain your reasoning.
8. How can you solve inequalities? Explain the process.

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply. 21st Century Themes		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* 21st Century Skills	
X	Global Awareness	E, T, A	Critical Thinking & Problem Solving
	Environmental Literacy	E, T	Creativity and Innovation
X	Health Literacy	E	Collaboration, Teamwork and Leadership
	Civic Literacy	E	Cross-Cultural and Interpersonal Communication
	Financial, Economic, Business and Entrepreneurial Literacy	E	Communication and Media Fluency
		E,T	Accountability, Productivity and Ethics

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

E	CRP1. Act as a responsible and contributing citizen and employee
E,T, A	CRP2. Apply appropriate academic and technical skills
E	CRP3. Attend to personal health and financial well-being

E,T	**CRP4. Communicate clearly and effectively with reason**
E	**CRP5. Consider the environmental, social and economic impacts of decisions**
E, T	**CRP6. Demonstrate creativity and innovation**
E, T, A	**CRP7. Employ valid and reliable research strategies**
E,T, A	**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them**
E	**CRP9. Model integrity, ethical leadership and effective management**
E	**CRP10. Plan education and career paths aligned to personal goals**
E	**CRP11. Use technology to enhance productivity**
E	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
1. The basic process/steps for simplifying expressions 2. The process/steps for solving equations and inequalities 3. The parts of a coordinate plane in order to graph inequalities (Ex: origin, x-axis, y-axis, etc…) 4. The differences between rational & irrational numbers (advanced only)	**College Prep**: 1. Write expressions using addition, subtraction, multiplication and division 2. Simplify expressions using Order of Operations and the Distributive Property 3. Solve multi-step equations 4. Solve word problems that involving rates, ratios, and convert units 5. Solve and apply proportions to word problems 6. Apply the use of proportions to solve for a missing side in

	similar figures 7. Solve multi-step inequalities and graph **Advanced**: The above CP understandings plus the following: 8. Use properties of rational and irrational numbers such as: simplify square root expressions, graph real numbers on a number line, simplify real numbers by adding, subtracting, multiplying, & dividing; & identify numbers as either rational or irrational 9. Solving percent's using proportions and percent equations 10. Find the percent of change in a word problem
Assessment Evidence:	
Performance Tasks: Students demonstrate their understanding of the standards through multi-step higher order thinking problems. Teachers will incorporate at least two of the following questions/activities from different standards into their instruction from illustrativemathematics.org. These tasks will be used as additional support when covering the material in the unit. Questions similar to the problems should be incorporated into formal assessments. Students should receive feedback from the teacher on their performance on Performance Tasks. Teachers can also use the Common Core Performance task at the beginning of each chapter in the Algebra 1 Common Core textbook by Pearson. College Prep: A.REI.B.3, A.REI.A.1 Reasoning with linear inequalities A.CED.A.4 Equations and Formulas	**Other Assessment Measures*:*** Quizzes, tests, academic prompts, observations, homework, benchmarks, journals, review games, & wipe board activities ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.***

A.SSE.A.1 Kitchen Floor Tiles A.CED.A.1 Planes and wheat A-CED.A.1 Paying the rent A.REI.A.1 Zero Product Property 1 A.CED.A.2 Clea on an Escalator Advanced: Any two of the above Performance Tasks plus the following problems: N.Q.A.1 Runners' World N.Q.A.2 Giving Raises N.Q.A.3 Calories in a Sports Drink	

Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?

Instructional Strategies and Activities *D*	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Algebra 1 enVision textbook by Pearson. If using another resource, please align content to what is identified in this plan.*** ***All teachers should be teaching the following lessons:*** 1. Expressions- Simplifying expressions using the order of operations and the Distributive property 2. Solving Equations- Solving and reasoning of one-step equations, two-step equations, multi-step equations, & equations with variables on both sides of the equal sign. 3. Proportions- How to solve using proportions and using proportions to solve for a missing side of similar figures. 4. Solving Inequalities- Solving and graphing inequalities using addition, subtraction, multiplication, and Division, plus inequalities that involve multiple steps

	Advanced teachers should also teach the following lessons: 5. Real Numbers- Addition, subtraction, multiplication, and division of real numbers. 6. Percents- Find the percent of change in word problems Activities at all levels should include but not limited to: review games (such as jeopardy, zonk, the big wheel, etc.), guided notes, guided practice problems, group work, higher-order thinking problems, activator at the beginning of each lesson, pass-the-problem activity, wipe boards, and closing activity/problem
Resources	
envision Algebra 1 Textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & EngageNY https://www.engageny.org/	
Suggested Time Frame:	CP: 58-60 days; Adv: 50-52 days

Content Area:	Algebra 1	Grade(s): 9-12
Unit Plan Title:	Unit 2: Introduction to Functions, Linear Functions, & Exponents and Exponential Functions	

Domain(s)

College Prep (CP): A.REI(Reasoning with Equations & Inequalities), A.CED(Creating Equations), N-RN(The Real Number System), F.IF(Interpreting Functions), & F.LE(Linear and Exponential Models)

Advance(ADV): all CP standards plus N.Q (Quantities), G-GPE(Expressing Geometric Properties with Equations), S-ID(Interpreting Categorical & Quantitative Data), F-BF(Building Functions)

Overview/Rationale

Equations & Inequalities plus Creating Equations: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Building Functions, Interpreting Functions and Linear & Exponential Models: Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

The Real Number Systems: In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by imaginary numbers to form complex numbers.

Expressing Geometric Properties with Equations: The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Quantities: Quantities has a real application to real world problems. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Interpreting Categorical & Quantitative Data: Data are gathered, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measure of shape, center, and spread. The shape of the distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range).

Content Standard(s) (Established Goals)

College Prep:

F-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y=f(x)*.

F-IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F-IF.B.4: For a functions that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **

F-IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

F-IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

S-ID.C.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

N-RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer's exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3=5(^{1/3})^3$ to hold, so $(5^{1/3})^3$ must equal 5.*

A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes and labels and scales.

A-CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Adv: All CP standards plus

N-Q.A.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

S-ID.B.6a: Fit a function to the data (including the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

S-ID.B.6b: Informally address the fit of a function by plotting and analyzing residuals, including with the use of technology.

S-ID.B.6c: Fit a linear function for a scatter plot that suggests a linear association.

S-ID.C.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID.C.9: Distinguish between correlation and causation.

N-RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

F-BF.A.1: Write a function that describes a relationship between two quantities.

F-BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

F-BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-IF.C.7a: Graph linear and quadratic function and show intercepts, maxima, and minima.

F-IF.C.7b: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F-LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE.A.1a: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F-LE.A.1c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

A-SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity.

A-SSE.B.3c: Use the properties of exponents to transform expressions for exponential functions.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments & critique the reasoning of others.
MP.4 Model with mathematics.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.
8.1.5.A.3: Use graphic organizers to organize information about problem or issue.
8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.
8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.
8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.
8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.
8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.
8.2.2.E.1: List and demonstrate the steps to an everyday task.

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

CP:

1. Interpret functions that arise in applications in terms of the context
2. Analyze functions using different representations
3. Understand the concept of a function and use function notation
4. Represent and solve equations and inequalities graphically
5. Construct and compare linear and exponential models and solve
6. Create equations that describe numbers or relationships
7. Extend the properties of exponents to rational exponents

Adv: The above CP understandings plus the following:

8. Reason Quantitatively and use units to solve problems
9. Use coordinates to prove simple geometric theorems algebraically
10. Summarize, represent, and interpret data on two categorical and quantitative variables
11. Build new functions from existing functions
12. Build a function that models a relationship between two quantities

Essential Question(s) : (What provocative questions will foster inquiry, understanding, and transfer of learning?)

1. How can you represent and describe functions?
2. Can functions describe real-world situations? Explain why or why not.
3. What does the slope of a line indicate about a line?
4. What information does the equation of a line give you?
5. How can you make predictions based on a scatter plot?
6. How can you represent numbers less than 1 using exponents?
7. How can you simplify expressions involving exponents?
8. What are the characteristics of exponential functions?

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply.			*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*	
21st Century Themes			**21st Century Skills**	
	X	**Global Awareness**	**E,T,A**	**Critical Thinking & Problem Solving**
		Environmental Literacy	**E,T**	**Creativity and Innovation**
	X	**Health Literacy**	**E,T**	**Collaboration, Teamwork and Leadership**
		Civic Literacy	**E**	**Cross-Cultural and Interpersonal Communication**
		Financial, Economic, Business and Entrepreneurial Literacy	**E,T**	**Communication and Media Fluency**
			E,T	**Accountability, Productivity and Ethics**

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

	E,T	**CRP1. Act as a responsible and contributing citizen and employee**
	E,T,A	**CRP2. Apply appropriate academic and technical skills**
	E,T	**CRP3. Attend to personal health and financial well-being**
	E,T,A	**CRP4. Communicate clearly and effectively with reason**
	E	**CRP5. Consider the environmental, social and economic impacts of decisions**
	E,T	**CRP6. Demonstrate creativity and innovation**
	E,T	**CRP7. Employ valid and reliable research strategies**
	E,T,A	**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them**
	E,T	**CRP9. Model integrity, ethical leadership and effective management**
	E	**CRP10. Plan education and career paths aligned to personal goals**

	E,T	**CRP11. Use technology to enhance productivity**
	E	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
1. Similarities and differences between linear and nonlinear functions 2. How to graph a linear equation 3. The slope-intercept and standard form formulas for linear equations 4. Properties of exponents 5. Describe what slope means and identify the four types of slope	CP: 1. Use graphs to relate two quantities 2. To identify and represent patterns that describe linear & nonlinear functions 4. Graph a function rule 5. Extend, identify, and write Arithmetic Sequences 6. Find the rate of change 7. Find the slope given a graph or two order pairs 8. Write and graph an equation of direct variation 9. Write linear equations using slope-intercept form & standard form 10. Simplify expressions involving zero and negative exponents 11. Properties of exponents: multiply powers with the same base, power to a power, product to a power, and dividing exponents Adv: The above CP understandings plus the following: 12. Write a function rule 13. Use point slope for linear equations 14. Write equations of parallel and perpendicular lines 15. Identify key information in a scatter plot and describe trends 16. Graph Absolute Value Functions 17. Rewrite expressions involving radicals and rational exponents 18. Evaluate and graph exponential functions

	19. Model exponential growth and decay 20. Write and use recursive formulas for geometric sequences
Assessment Evidence:	
Performance Tasks: Students demonstrate their understanding of the standards through multi-step higher order thinking problems. Teachers will incorporate at least two of the following questions/activities from different standards into their instruction from illustrativemathematics.org. More questions are located at illustrative mathematics website. These tasks will be used as additional support when covering the material in the unit. Questions similar to the problems should be incorporated into formal assessments. Students should receive feedback from the teacher on their performance on Performance Tasks. Teachers can also use the Common Core Performance task at the beginning of each chapter in the Algebra 1 Common Core textbook by Pearson. College Prep: F.IF.A.1 The Parking Lot F.IF.A.2 Yam in the Oven F.IF.B.4 Warming and Cooling F.IF.B.4, F.IF.B.5 Average Cost F.LE.B.5 US Population 1982-1988 F.IF.B.6 Temperature Change	**Other Assessment Measures*:*** Quizzes, tests, academic prompts, observations, homework, benchmarks, journals, review games, & wipe board activities ******All benchmarks will be collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.***

Advanced: Any two of the above Performance Tasks plus the following problems: F.LE.A.1 Finding Linear and Exponential Models F.LE.A.2 Interesting Interest Rates F.IF.C.7b Bank Account Balance	

Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?

Instructional Strategies and Activities *D*	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Algebra 1 enVision textbook by Pearson. If using another resource, please align content to what is identified in this plan.*** ***All teachers should be teaching the following lessons:*** 1. Introduction to functions- Includes solving in interpreting linear and nonlinear functions plus identifying and interpreting patterns and arithmetic sequences 2. Linear Functions- Understand what slope means and be able to identify the rate of change within a given slope. Use the slope-intercept and standard form when writing and graphing a linear equation. 3. Exponents- Understand the result of a zero or negative exponent. Apply the rules for multiplying powers with the same base, power to power, power to products and dividing exponents Advanced teachers should also teach the following lessons: 4. Rules of exponential functions including growth and decay 5. Write a function rule for a given situation 6. Use standard form to write and graph a linear equation Activities at all levels should include but not limited to: review games (such as jeopardy, zonk, the big wheel, etc.), guided notes, guided practice problems, group work, higher-order thinking problems, activator at the beginning of each lesson, pass-the-problem activity, wipe boards, and closing activity/problem

Resources

envision Algebra 1 Textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & EngageNY https://www.engageny.org/

Content Area:	Algebra 1	Grade(s): 9-12
Unit Plan Title:	Unit 3: Systems of equations, Polynomials & Factoring, Quadratic Functions & Equations, and Rational Expressions & Functions	

Domain(s)

CP: A-REI(Reasoning with Equations & Inequalities),A-APR(Arithmetic with Polynomials & Rational Expressions), A-SSE(Seeing Structure in Expressions), A-CED.(Creating Equations), F.IF(Interpreting Functions)

Adv: All the above CP standards plus F-LE.(Linear and Exponential Models)

Overview/Rationale

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

Equations & Inequalities plus Creating Equations: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Building Functions, Interpreting Functions and Linear & Exponential Models: Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Content Standard(s) (Established Goals)

College Prep:

A-REI.B.4a: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula.

A-REI.B.4b: Solve quadratics equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A-REI.D.11:

Explain why the x-coordinates of the points where the graphs of the equations y= f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

A-REI.D.12: Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

A-APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.*

A.SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficient.

A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it.

A-SSE.B.3a: Factor a quadratic expression to reveal the zeros of the function it defines.

A-SSE.B.3b: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

A-CED.A.1: Create equations and inequalities in one variable and use to solve problems, Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Adv: All the above CP standards plus

A-REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solution.

A-REI.C.6: Solve systems of linear equations exactly and appropriately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A-REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variable algebraically and graphically. *For example, find the points of intersection between the line $y=-3x$ and the circle $x^2+y^2=3$.*

F-IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.B.4: For a functions that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **

F-IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

F-IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F-IF.C.7a: Graph linear and quadratic function and show intercepts, maxima, and minima.
F-IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF.C.8a: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
F-IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F-BF.A.1: Write a function that describes a relationship between two quantities.
F-BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes and labels and scales.
A-CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
S-ID.B.6a: Fit a function to the data (including the use of technology); use functions fitted to data to solve problems in the context of the data. Uses given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
S-ID.B.6b: Informally address the fit of a function by plotting and analyzing residuals, including with the use of technology.
F-LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
A-APR.B.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
A-SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments & critique the reasoning of others.
MP.4 Model with mathematics.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeating reasoning.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.
8.1.5.A.3: Use graphic organizers to organize information about problem or issue.
8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.
8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.
8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.
8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.
8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.
8.2.2.E.1: List and demonstrate the steps to an everyday task.

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

Students will understand…
CP:
1. Representing and solving equations and inequalities graphically
2. Solving equations and inequalities in one variable
3. Performing arithmetic operations on polynomials
4. Interpreting the structure of expressions

5. Creating equations that describe numbers or relationships
6. Interpreting functions that arise in applications in terms of the context

Adv: The above CP understandings plus the following:
7. Understand the concept of a function and use function notation
8. Analyzing functions using different representations
9. Solving systems of equations

Essential Question(s) : (What provocative questions will foster inquiry, understanding, and transfer of learning?)

1. How can you solve a system of equations or inequalities?
2. Can systems of equations model real-world situations? Explain your answer.
3. Can two algebraic expressions that appear to be different be equivalent? Explain your answer.
4. How are properties of real numbers related to polynomials?
5. What are the characteristics of quadratic functions?
6. How can you solve a quadratic equation?
7. How can you use functions to model real-life situations?

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply.

21st Century Themes

X	**Global Awareness**
	Environmental Literacy
X	**Health Literacy**
	Civic Literacy
	Financial, Economic, Business and

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

21st Century Skills

E, T, A	**Critical Thinking & Problem Solving**
E, T	**Creativity and Innovation**
E	**Collaboration, Teamwork and Leadership**
E	**Cross-Cultural and Interpersonal Communication**
E	**Communication and Media Fluency**

	Entrepreneurial Literacy	E,T	Accountability, Productivity and Ethics

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

E	**CRP1. Act as a responsible and contributing citizen and employee**
E,T, A	**CRP2. Apply appropriate academic and technical skills**
E	**CRP3. Attend to personal health and financial well-being**
E,T	**CRP4. Communicate clearly and effectively with reason**
E	**CRP5. Consider the environmental, social and economic impacts of decisions**
E, T	**CRP6. Demonstrate creativity and innovation**
E, T, A	**CRP7. Employ valid and reliable research strategies**
E,T, A	**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them**
E	**CRP9. Model integrity, ethical leadership and effective management**
E	**CRP10. Plan education and career paths aligned to personal goals**
E	**CRP11. Use technology to enhance productivity**
E	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
Both CP & Adv.: 1. How to graph a linear inequality and equation 2. The basic steps for factoring polynomials including binomials and trinomials 3. Identify and apply the Quadratic formula	CP: 1. Graph linear inequalities in two variables. 2. Add, subtract, multiply and factor polynomials 3. Multiply binomials

4. Solving systems of equation by either one of the three methods will give you the same answer.	4. Factor trinomials in the form x^2+bx+c and ax^2+bx+c 5. Solving and factoring Quadratic equations 6. Solve rational equations 7. Graph rational functions Adv: The above CP understandings plus the following: 8. Solve systems of equations by graphing, elimination method, or substitution method 9. Solving systems of linear and quadratic equations
Assessment Evidence:	
Performance Tasks: Students demonstrate their understanding of the standards through multi-step higher order thinking problems. Teachers will incorporate at least two of the following questions/activities from different standards into their instruction from illustrativemathematics.org. More questions are located at illustrative mathematics website. These tasks will be used as additional support when covering the material in the unit. Questions similar to the problems should be incorporated into formal assessments. Students should receive feedback from the teacher on their performance on Performance Tasks. Teachers can also use the Common Core Performance task at the beginning of each chapter in the Algebra 1 Common Core textbook by Pearson. College Prep: A.REI.D.12 Fishing Adventures 3 A.APR.A.1 Powers of 11 A.SSE.A.1 Mixing Candies A.REI.B.4 Visualizing Completing the Square	**Other Assessment Measures*:*** Quizzes, tests, academic prompts, observations, homework, benchmarks, journals, review games, & wipe board activities ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.***

A.REI.B.4 Braking Distance A.REI.B.4 Two Squares are Equal F.IF.B.4 Words – Tables - Graphs F.IF.B.5 The restaurant F.IF.B.4 The Aquarium F.IF.B.4 Containers F.IF.B.4-5 The Canoe Trip, Variation 2 Advanced: Also complete the two problems below: A.REI.C.5 Solving Two Equations in Two Unknowns F.LE.A.1 Finding Linear and Exponential Models	

Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?

Instructional Strategies and Activities *D*	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Algebra 1 enVision textbook by Pearson. If using another resource, please align content to what is identified in this plan.*** ***All teachers should be teaching the following lessons:*** 1. Graphing linear equations and inequalities 2. Operations of Polynomials (Ex: add, subtract, multiply polynomials; factoring polynomials) 3. Solve Quadratic Equations (factoring, completing the square & quadratic formula) Advanced teachers should also teach the following lessons: 4. Solving systems of equations and inequalities 5. Solve Quadratic Functions by graphing 6. Dividing Polynomials 7. Rational Expressions (simplifying, multiplying, & dividing)

Resources	
envision Algebra 1 Textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & Engage NY https://www.engageny.org/	
Suggested Time Frame:	CP: 58-60 days Adv: 49-51 days

Content Area:	Algebra 1	Grade(s): 9-12
Unit Plan Title:	Unit 4: Radical Expressions and Equations plus Data Analysis and Probability	

Domain(s)

Advanced Only: A.REI(Reasoning with Equations & Inequalities), F.IF(Interpreting Functions), S-ID(Interpreting Categorical & Quantitative Data), A.APR(Arithmetic with Polynomials and Rational Expressions)

Overview/Rationale

Reasoning with Equations & Inequalities: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Interpreting Functions : Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

The Real Number Systems: In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by imaginary numbers to form complex numbers.

Quantities: Quantities has a real application to real world problems. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Content Standard(s) (Established Goals)

F-IF.B.4: For functions that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts;*

intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

F-IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.C.7b: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-BF.A.1b: Combine standard function types using arithmetic operations.

F-BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF.B.4: Find inverse functions.

F-BF.B.4a: Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.

S.ID.A.1: Represent data on the real number line (dot plots, histograms, and box plots).

S.ID.A.2: Use statistic appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-ID.A.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S-IC.B.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments & critique the reasoning of others.
MP.4 Model with mathematics.
MP.5 Use appropriate tools strategically.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.
8.1.5.A.3: Use graphic organizers to organize information about problem or issue.
8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.
8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.
8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.
8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.
8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.
8.2.2.E.1: List and demonstrate the steps to an everyday task.

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

1. Simply radicals and radical expressions
2. Graph square root functions
3. Use a variety of ways to organize and represent Data(Ex: Matrix, histogram, Box-&-Whisker plot)
4. Permutations & Combinations
5. The difference between Theoretical and Experimental Probability

Essential Question(s) : (What provocative questions will foster inquiry, understanding, and transfer of learning?)

1. How are radical expressions represented?
2. What are the characteristics of square root functions?
3. How can you solve a radical equation?
4. How can collecting and analyzing data help you make decisions or predictions?
5. How can you make and interpret different representations of data?
6. How is probability related to real-world events?

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply. 21st Centur y Themes		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* 21st Centur y Skills	
X	Global Awareness	E, T, A	Critical Thinking & Problem Solving
	Environmental Literacy	E, T	Creativity and Innovation
	Health Literacy	E, T	Collaboration, Teamwork and Leadership
	Civic Literacy	E, T	Cross-Cultural and Interpersonal Communication
X	Financial, Economic, Business and Entrepreneurial Literacy	E,T	Communication and Media Fluency
		E, T	Accountability, Productivity and Ethics

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

E, T	CRP1. Act as a responsible and contributing citizen and employee
E, T, A	CRP2. Apply appropriate academic and technical skills
E, T	CRP3. Attend to personal health and financial well-being
E, T	CRP4. Communicate clearly and effectively with reason
E, T	CRP5. Consider the environmental, social and economic impacts of decisions
E, T	CRP6. Demonstrate creativity and innovation

	E, T	**CRP7. Employ valid and reliable research strategies**
	E, T	**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them**
	E, T	**CRP9. Model integrity, ethical leadership and effective management**
	E, T	**CRP10. Plan education and career paths aligned to personal goals**
	E, T	**CRP11. Use technology to enhance productivity**
	E, T	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
1. The basic terms of probability 2. The difference between quantitative and qualitative data 3. The difference between a Permutation and Combination 4. Basic operations with radicals(addition, subtraction, multiplication, & Division) 5. Pythagorean Theorem 6. How to solve equations and inequalities	1. Interpret categorical and Quantitative Data 2. Make inferences and justify conclusions 3. Conditional Probability and the Rules of Probability 4. Display and analyze data in a matrix, frequency table, histogram, and box-and-whisker 5. Solve equations with radicals 6. Use trigonometric ratios to find a side length of a right triangle

Assessment Evidence:

Performance Tasks: Students demonstrate their understanding of the standards through multi-step higher order thinking problems. Teachers will incorporate at least two of the following questions/activities from different standards into their instruction from illustrativemathematics.org. These tasks will be used as additional support when covering the material in the unit. Questions similar to the problems should be incorporated into formal assessments. Students should receive feedback from the teacher on their	**Other Assessment Measures*:*** Quizzes, tests, academic prompts, observations, homework, benchmarks, journals, review games, & wipe board activities ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.***

performance on Performance Tasks. Teachers can also use the Common Core Performance task at the beginning of each chapter in the Algebra 1 Common Core textbook by Pearson. F.IF.C.7b Bank Account Balance S.ID.A.1-3 Haircut Costs S.ID.A.1-3 Speed Trap S.ID.A.2-3 Measuring Variability in a Data Set S.ID.A.3 Identifying Outliers S.ID.B.5 Support for a Longer School Day? S.ID.B.6 Laptop Battery Charge 2 F.IF.B.4 The Aquarium F.IF.B.4 Containers F.IF.B.4-5 The Canoe Trip, Variation 2	

Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?

Instructional Strategies and Activities *D*	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Algebra 1 enVision textbook by Pearson. If using another resource, please align content to what is identified in this plan.*** **The following lessons will be taught:** 1. Right Triangles- Use Pythagorean Thereon and Trigonometric ratios to find a missing leg or hypotenuse. Also use Trigonometric ratios to find an angle measure 2. Radical Expressions- Simplify radicals involving products and quotients. Then teach students how to simplify radical expressions involving addition, subtraction, multiplication, and division. 3. Radical Equations- Solve equations containing radicals and identify extraneous solutions. 4. Square Root Functions- Graph square root functions and translate graphs of square roots.

	5. Data Displays- Create and interpret data using Matrices, frequency tables, histograms, and box-and-whisker plots. 6. Matrices- Add and subtract matrices plus multiply a matrix by a scalar. 7. Permutations & Combinations- Teach the formulas for Permutations & Combinations and accurately use them when appropriate. Incorporate the Multiplication Counting Principal. Find the probabilities of mutually exclusive and overlapping events.
Resources	
envision Algebra 1 Textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & Engage NY https://www.engageny.org/	
Suggested Time Frame:	25 days

Curricular Map of High School Student Learning Standards for Mathematics

	Algebra	Geometry	Algebra II	Pre calculus
NUMBER AND QUANTITY (N)				
The Real Number System (RN)				
N-RN.A.1.Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	X		X	X
N-RN.A.2.Rewrite expressions involving radicals and rational exponents using the properties of exponents.	X		X	X
N-RN.B.3.Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	X			X
Quantities (Q)				
N-Q.A.1.Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	X	X		X
N-Q.A.2.Define appropriate quantities for the purpose of descriptive modeling.	X	X	X	X
N-Q.A.3.Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	X			X
The Complex Number System (CN)				
N-CN.A.1.Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real.			X	
N-CN.A.2.Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.			X	
N-CN.A.3.(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.			X	X

	Algebra	Geometry	Algebra II	Pre calculus
N-CN.B.4.(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.				X
N-CN.B.5.(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.				X
N-CN.B.6.(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				X
N-CN.C.7.Solve quadratic equations with real coefficients that have complex solutions.			X	
N-CN.C.8.(+) Extend polynomial identities to the complex numbers.			X	X
N-CN.C.9.(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.			X	X
Vector and Matrix Quantities (VM)				
N-VM.A.1.(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v, 14 114,** v).		X		X
N-VM.A.2.(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				X
N-VM.A.3.(+) Solve problems involving velocity and other quantities that can be represented by vectors.				X

	Algebra	Geometry	Algebra II	Pre calculus
N-VM.B.4. (+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction v— was v + (—w), where —w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.		X		
N-VM.B.5. (+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vv) = (mix, cv_i). b. Compute the magnitude of a scalar multiple cv using licv11= iciv. Compute the direction of cv knowing that when Iclv # 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).				X
N-VM.C.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.				X
N-VM.C.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.				X
N-VM.C.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.				X
N-VM.C.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.				X
N-VM.C.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.				X

	Algebra	Geometry	Algebra II	Pre calculus
N-VM.C.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.				X
N-VM.C.12. (+) Work with 2 x 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.				X
Algebra (A)				
Seeing Structure in Expressions (SSE)				
A-SSE.A.1. Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	X	X	X	X
A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it.	X		X	X
A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions.	X	X	X	X
A-SSE.B.4. Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.			X	X
Arithmetic with Polynomials and Rational Expressions (APR)				
A-APR.A.1.Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	X		X	
A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	X		X	X

	Algebra	Geometry	Algebra II	Pre calculus
A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	X		X	X
A-APR.C.4. Prove polynomial identities and use them to describe numerical relationships.			X	
A-APR.C.5. (+) Know and apply the Binomial Theorem for the expansion of *(x* + y)" in powers of *x* and y for a positive integer *n,* where *x* and y are any numbers, with coefficients determined for example by Pascal's Triangle.			X	
A-APR.D.6.Rewrite simple rational expressions in different forms; write $a^{(}x^{)}/_{b(x)}$ in the form *q(x)* + $^{trx)}$/ *b(x),* where a(x), *b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x),* using inspection, long division, or, for the more complicated examples, a computer algebra system.	X		X	
A-APR.D.7.(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	X		X	
Creating Equations (CED)				
A-CED.A.1.Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	X			
A-CED.A.2.Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	X	X		
A-CED.A.3.Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	X		X	
A-CED.A.4.Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	X			X

	Algebra	Geometry	Algebra II	Pre calculus
Reasoning with Equations and Inequalities (REI)				
A-REI.A.1.Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**X**		**X**	**X**
A-REI.A.2.Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**X**		**X**	**X**
A-REI.B.3.Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**X**			**X**
A-REI.B.4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b.*	**X**		**X**	**X**
A-REI.C.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**X**			
A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	**X**		**X**	
A-REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.			**X**	
A-REI.C.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.				**X**
A-REI.C.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).				**X**

	Algebra	Geometry	Algebra II	Pre calculus
A-REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	X			
A-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y= *f(x)* and y = *g(x)* intersect are the solutions of the equation *f(x) = g(x);* find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	X	X		X
A-REI.D.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	X			
FUNCTIONS (F)				
Interpreting Functions (IF)				
F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x.* The graph of *f* is the graph of the equation y= *f(x).*	X			X
F-IF.A.2. Use function notations, evaluates functions for inputs in their domains, and interprets statements that use function notation in terms of a context.	X			X
F-IF.A.3.Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	X		X	
F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	X			X
F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it descrbes.*	X			X

	Algebra	Geometry	Algebra II	Pre calculus Math Topics
F-IF.B.6.Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	X		X	X
F-IF.C.7.Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step *functions* and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are *available,* and showing end behavior. *d.* (+) Graph rational functions, identifying zeros and asymptotes when suitable *factorizations* are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end *behavior,* and trigonometric functions, showing period, midline, and amplitude.	X		X	X
F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.	X		X	
F-IF.C.9.Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	X		X	
Building Functions (BF)				
F-BF.A.1.Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. c. (+) Compose functions.	X		X	X
F-BF.A.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	X		X	X

	Algebra	Geometry	Algebra II	Pre calculus
F-BF.B.3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	X		X	X
F-BF.B.4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.			X	X
F-BF.B.5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving exponents and logarithms.			X	X

	Algebra	Geometry	Algebra II	Pre calculus
Linear, Quadratic and Exponential Models (LE)				
F-LE.A.1.Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	X			X
F-LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	X		X	
F-LE.A.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	X			
F-LE.A.4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e;* evaluate the logarithm using technology.			X	X
F-LE.B.5.Interpret the parameters in a linear or exponential function in terms of a context.	X			
Trigonometric Functions (TF)				
F-TF.A.1.Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.			X	X
F-TF.A.2.Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.			X	X
F-TF.A.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for Tr13, Tr/4 and Tr/6, and use the unit circle to express the values of sine, cosines, and tangent for *x,* IF + *x,* and 2¯rr — *x* in terms of their values for *x,* where *x* is anyreal number.				X

	Algebra	Geometry	Algebra H	Pre calculus
F-TF.A.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.				X
F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*			X	X
F-TF.B.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.				X
F-TF.B.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*				X
F-TF.C.8. Prove the Pythagorean identity $\sin^2(0) + \cos^2(0) = 1$ and use it to find sin(0), cos(e), or tan(0) given sin(0), cos(0), or tan(0) and the quadrant of the angle.			X	X
F-TF.C.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.				X
GEOMETRY (G)				
Congruence (CO)				
G.CO.A.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.		X		
G-CO.A.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).		X		
G-CO.A.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.		X		
G-CO.A.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		X		
G-CO.A.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.		X		
G-CO.B.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.		X		
G-CO.B.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.		X		

	Algebra	Geometry	Algebra II	Pre calculus
G-CO.B.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.		X		
G-CO.C.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*		**X**		**X**
G-CO.C.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*		**X**		**X**
G-CO.C.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*		**X**		
G-CO.D.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*		**X**		
G-CO.D.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.		**X**		
Similarity, Right Triangles, and Trigonometry (SRT)				
G-SRT.A.1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.		**X**		
G-SRT.A.2.Given two figures use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.		X		
G-SRT.A.3.Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	**X**	**X**		
G-SRT.B.4.Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*		**X**		

	Algebra	Geometry	Algebra II	Pre calculus
G-SRT.B.5.Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.		X		X
G-SRT.C.6.Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		X	X	X
G-SRT.C.7.Explain and use the relationship between the sine and cosine of complementary angles.		X		X
G-SRT.C.8.Use trigonometric ratios and Pythagorean Theorem to solve right triangles in applied problems.*	X	X		X
G-SRT.D.9.(+) Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.		X		X
G-SRT.D.10.(+) Prove the Laws of Sines and Cosines and use them to solve problems.		X		X
G-SRT.D.11.(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).		X		X
Circles (C)				
G-C.A.1.Prove that all circles are similar.		X		
G-C.A.2.Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*		*X*		
G-C.A.3.Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.		X		
G-C.A.4.(+) Construct a tangent line from a point outside a given circle to the circle.		X		
G-C.B.5.Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.		X		
Expressing Geometric Properties with Equations (GPE)				
G-GPE.A.1.Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		X		X
G-GPE.A.2.Derive the equation of a parabola given a focus and directrix.			X	X
G-GPE.A.3.(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	X			X

	Algebra	Geometry	Algebra II	Pre calculus
G-GPE.B.4.Use coordinates to prove simple geometric theorems algebraically.		X	X	
G-GPE.B.5.Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	X	X		
G-GPE.B.6.Find the point on a directed line segment between two given points that partitions the segment in a given ratio.		X		
G-GPE.B.7.Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*				
Geometric Measurement and Dimension (GMD)				
G-GMD.A.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*		X		X
G-GMD.A.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.		X		X
G-GMD.A.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*		X		X
G-GMD.B.4. Identify the shape of two-dimensional cross-sections of three-dimensional objects, and identifies three-dimensional objects generated by rotations of two-dimensional objects.		X		
Modeling with Geometry (MG)				
G-MG.A.1.Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*		X		X
G-MG.A.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*				
G-MG.A.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*		X		
Statistics and Probability (S)				
Interpreting Categorical and Quantitative Data (ID)				
S-ID.A.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	X			

	Algebra	Geometry	Algebra II	Pre-calculus
S-ID.A.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	X			X
S-ID.A.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	X			X
S-ID.A.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	X			X
S-ID.B.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	X			X
S-ID.B.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data (including with the use of technology) to solve problems in the context of the data. Uses given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology c. Fit a linear function for a scatter plot that suggests a linear association.	X			X
S-ID.C.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	X			
S-ID.C.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	X			
S-ID.C.9. Distinguish between correlation and causation.	X			
Making Inferences and Justifying Conclusions (IC)				
S-IC.A.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.			X	X

	Algebra	Geometry	Algebra II	Pre calculus
S-IC.A.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.			X	X
S-IC.B.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.			X	
S-IC.B.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.			X	X
S-IC.B.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.			X	
S-IC.B.6. Evaluate reports based on data.			X	
Conditional Probability and the Rules of Probability (CP)				
S-CP.A.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").			X	X
S-CP.A,2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.			X	X
S-CP.A.3. Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B),* and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A,* and the conditional probability of *B* given *A* is the same as the probability of *B.*			X	X
S-CP.A.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.			X	
S-CP.A.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.			X	X
S-CP.B.6. Find the conditional probability of *A* given *B* as the fraction of B's outcomes that also belong to *A,* and interpret the answer in terms of the model.			X	X
S-CP.B.7. Apply the Addition Rule, P(A or B) = P(A) + P(B) — P(A and B), and interpret the answer in terms of the model.			X	X
S-CP.B.8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(BIA) = P(B)P(AIB), and interpret the answer in terms of the model.				X

	Algebra	Geometry	Algebra II	Pre calculus
S-CP.B.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.				X
Using Probability to Make Decisions (MD)				
S-MD.A.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.				
S-MD.A.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.				
S-MD.A.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.				
S-MD.A.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.				
S-MD.B.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. b. Evaluate and compare strategies on the basis of expected values.				
S-MD.B.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).				
S-MD.B.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).				

APPENDIX A: Webb's Depth-of-Knowledge (DOK) Levels for Mathematics created by Norman Webb

According to Norman L. Webb, Wisconsin Center for Educational Research ("Depth-of-Knowledge Levels for Four Content Areas," March 22, 2002), "interpreting and assigning depth-of-knowledge levels to both objectives within standards and assessment items is an essential requirement of alignment analysis. Four levels of depth-of-knowledge are used for this analysis." Norman Webb's "Depth-of-Knowledge Levels for four Content Areas" include: Language Arts (reading, Writing), Mathematics, Science, and Social Studies.

A general definition for each of the four (Webb) Depth-of-Knowledge levels is followed by Table 1, which provide further specifications and examples for each of the DOK levels. Webb recommends that large scale, on-demand assessments in mathematics should only assess Depth of Knowledge Levels 1, 2, and 3. Depth of Knowledge at Level 4 in mathematics should be reserved for local assessment.

Level 1 (Recall) includes the recall of information such as fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify", "recall", "recognize", "use", and "measure". Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify", "organize", "estimate", "make observations", "collect and display data", and "compare data". These actions imply more than one step. For example, to compare data requires first identifying characteristics of the object or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain", "describe", or "interpret" could be classified at different levels depending on the object of the action. For example, if an item requires students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered

and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level of other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be complex. Students should be required to make several connections-relate ideas *within* the content area or *among* content areas-and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

Math Descriptors- Combined Webb Depth of Knowledge Levels of Mathematics (Webb, 2002), NAEP 2002 Mathematics Levels of Complexity, and Other Descriptors related to NECAP GLEs.

LEVEL 1 RECALL	LEVEL 2 SKILLS/CONCEPTS	LEVEL 3 STRATEGIC THINKING	LEVEL 4 EXTENDED THINKING
Examples represent, but do not constitute all Level 1 mathematics performances: Recall or recognize a fact, definitions, or terms Apply a well-known algorithm Apply a formula Determine the area or perimeter of rectangles or triangles given drawing and labels Identify a plan or three dimensional figure Measure a length Perform a specified or routine procedure Evaluate an expression	**Examples represent, but do not constitute all Level 2 mathematics performances:** Classify plane and three dimensional figures Interpret information from a simple graph Use models to represent mathematical concept Solve a **routine** problem requiring multiple steps, or the application of multiple concepts Compare figures or statements Provide justifications for steps in a solution process Extend a pattern	**Examples represent, but do not constitute all Level 3 mathematics performances:** Interpret information from a complex graph Explain thinking when more than one response is possible Make and/or justify conjectures Develop logical arguments for a concept Use concepts to solve problems Perform procedure with multiple steps and multiple decision points Generalize a pattern Describe, compare and contrast solution methods	**Examples represent, but do not constitute all Level 4 mathematics performances:** Relate mathematical concepts to other content areas Relate mathematical concepts to real-world applications in new situation Apply a mathematical model to illuminate a problem, situation Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results Design a mathematical model to inform and solve a practical or abstract situation ***NOTE:*** **Level 4 requires applying**

Solve a one-step word problem Retrieve information from a table or graph Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percent's), or within and between customary and metric measures Locate numbers on a number line, or points on a coordinate grid Solves linear equations Represent math relationships in words, pictures, or symbols	Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps Translate between tables, graphs, words and symbolic notation Select a procedure according to criteria and perform it	Formulate a mathematical model for a complex situation Provide mathematical justifications Solve a multiple step problem, supported with a mathematical explanation that justifies the answer Formulate an original problem, given a situation	**one approach among many to solve problems. Involves complex restructuring of data, establishing and evaluating criteria to solve problems.**

Lesson alignment of Pearson enVision Algebra 1 with MTHS Algebra 1 Curriculum

Color Coding: All Levels Advanced

UNIT 1 – Solving Linear Equations & Inequalities

Chapter 1

1-1 1-2 1-3 1-4 1-5 1-6 1-7

UNIT 2 – Introduction to Functions and Linear Functions & Exponents and Exponential Functions

Chapter 2

2-1 2-2 2-3 2-4

Chapter 3

3-1 3-2 3-3 3-4 3-5 3-6

Chapter 5

5-1 5-2 5-3 5-4

Chapter 6

6-1 6-2 6-3 6-4 6-5

UNIT 3 – Systems of Equations, Polynomials & Factoring, Quadratic Functions & Equations, and Rational Expressions & Functions

Chapter 4

4-1 4-2 4-3 4-4 4-5

Chapter 7

7-1 7-2 7-3 7-4 7-5 7-6 7-7

Chapter 8

8-1 8-2 8-3 8-4 8-5

Chapter 9

9-1 9-2 9-3 9-4 9-5 9-6 9-7

UNIT 4 – Radical Expressions and Equations plus Data Analysis and Probability

Chapter 10

10-1 10-2 10-3 10-4 10-5 10-6 10-7

Chapter 11

11-1 11-2 11-3 11-4 11-5

ALGEBRA II CURRICULUM

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Introduction

This document serves to meet all requirements for curriculum as per the Middle Township Board of Education and the New Jersey Department of Education and will serve as a guide for lesson planning. Units within the curricular framework for mathematics are designed to be taught in the order in which they are presented. There is a logical and developmentally-appropriate progression of standards, with strong consideration given to Major, Supporting, and Additional content standards presented since most concepts build upon each other. Within the units, the teachers have flexibility of what order to present the standards. Major, Supporting and Additional clusters of mathematics content standards are based on the New Jersey Student Learning Standards. Suggested Mathematical Practice Standards are listed in each unit to be imbedded regularly in daily math instruction.

Course Description

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x+1=0$ is an integer, not a whole number; the solution of $2x + 1= 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.
The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b2)/2)h$, can be solved for h using the same de Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same func tion. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on impo rtant "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). The eight standards for mathematical practices are as follows:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathemat ics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the

elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situation s, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a stud ent from engaging in the mathematical practices.

In this respect, those content standards, which set an expectation of understanding, are potential "points of intersection" b etween the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection a re intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professio nal development, and student achievement in mathematics.

Pacing Guide

UNIT TITLE	ENDURING UNDERSTANDINGS & KEY CONCEPTS	NJSLS	TIMEFRAME
Problem Solving in All Units	Understanding of what has occurred, why the process was effective, and can justify this with sound reasoning and evidence. Multiple strategies may be used to solve problems, although some are more effective than others. Reasonableness of solutions is determined by the context of the problems.	***Standards for Mathematical Practice MP1 through MP8***	Integrated Throughout Units/Course
1 - Linear Functions and Systems	• The language of mathematics is communicated through specialized vocabulary and symbols used to represent and describe mathematical ideas, generalizations, and relationships.	***All Levels:*** **A.CED.A.1 & 2** **F.BF.A.1 & 2** **F.IF.B.4** **F.LE.A.2** ***Adv. and Honors Only:*** **A.CED.A.3** **F.BF.B.3** **F.IF.A. 3 & B.5** **F.IF.C.7.b** **REI.C.6**	CP: 25-30 days Adv.: 22-25 days Honors: 20 days
2 - Quadratic Equations, Quadratic Functions, and Polynomial Functions	• The language of mathematics is communicated through specialized vocabulary and symbols used to represent and describe mathematical ideas, generalizations, and relationships. • Representing mathematical ideas involves using a variety of representations such as graphs, numbers, algebra,	***All Levels:*** **F.IF.B.4 & 6** **F.IF.C.7, 8 & 9** **A.SSE.A.2** **A.APR.A.1** **A.APR.B.2 & 3**	CP: 43-52 days Adv: 38-43 days Honors: 38 days

	words, and physical models to convey practical situations. • Mathematical problem solvers apply a variety of strategies and methods to solve problem situations.	**A.APR.C.4 & 5** **A.APR.D.6** **A.REI.D.11** **F.BF.A.1** **F.BF.B.3** **N.CN.C.8 & 9** **A.CED.A.2** ***Adv. and Hns. Only:*** **A.REI.B.4** **N.CN.A.1, 2, & 3** **N.CN.C.7**	
3 - Radical Functions, Rational Exponents, Radical Functions & Logarithmic Functions	• The language of mathematics is communicated through specialized vocabulary and symbols used to represent and describe mathematical ideas, generalizations, and relationships. • Representing mathematical ideas involves using a variety of representations such as graphs, numbers, algebra, words, and physical models to convey practical situations. • Mathematical problem solvers apply a variety of strategies and methods to solve problem situations.	***All Levels:*** **A.SSE.A.1a & b** **A.SSE.A.2** **A.SSE.B.3.c** **A.SSE.B.4** **A.REI.A.1 & 2** **A.REI.D.11** **A.CED.A.1, 2, & 4** **N.RN.A.1 & 2** **F.BF.A.1b & c** **F.BF.B.3** **F.IF.A.3** **F.IF.B.4, 5, & 6** **F.IF.C.7d & e** **F.IF.C.8 & 9** **F.LE.A.2** **F.LE.B.5**	CP: 60-69 days Adv: 47-56 days Honors: 42-45 days

		A.APR.D.6 & 7 ***Adv. and Hns. Only:*** **F.BF.B.4a & c** **F.IF.C.7b** **F.LE.A.4**	
4 – Probability, Data Analysis, and Statistics	• The language of mathematics is communicated through specialized vocabulary and symbols used to represent and describe mathematical ideas, generalizations, and relationships. • Representing mathematical ideas involves using a variety of representations such as graphs, numbers, algebra, words, and physical models to convey practical situations. • Mathematical problem solvers apply a variety of strategies and methods to solve problem situations.	***All Levels:*** **S.IC.A.1** **S.IC.B.3, 4, 5 & 6** ***Adv.and Hns. Only:*** **S.CP.A.1,2,3,4,5** **S.CP.B.6,7,8,9** **S.MD.B.6 & 7** **S.ID.A.4** **S.IC.A.2**	CP: 18-20 days Adv: 18-20 days Honors: 28-30 days

Content Area:	Algebra 2	Grade(s): 9-12
Unit Plan Title:	Unit 1 Linear Functions & Systems	

Domain(s)

A.CED; F.IF; F.BF; F.LE; A.REI

Overview/Rationale

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

Equations & Inequalities plus Creating Equations: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Building Functions, Interpreting Functions and Linear & Exponential Models: Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Interpreting Functions : Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

The Real Number Systems: In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by imaginary numbers to form complex numbers.

Quantities: Quantities has a real application to real world problems. In real world problems, the answers are usually not numbers but

quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Content Standard(s) (Established Goals)

College Prep

A.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

F.BF.A.1: Write a function that describes a relationship between two quantities.*

F.BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.IF.B.4: For a functions that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **

F.LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description, or two input-output pairs (include reading these from a table).

REI.D.11: Explain why the x-coordinates of the equation y=f(x) and y=g(x) intersects are the solutions of the equation f(x)=g(x); find the solutions approximately. E.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f0x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

Advanced and Honors

A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling content*. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

F.BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.IF.A.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0)=f91)=1, f(n+1)=f(n)+ F9n-1) for n≥1.

F.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate*

*domain for the function.**
F.IF.C.7.b: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.

8.1.5.A.3: Use graphic organizers to organize information about problem or issue.

8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.

8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.

8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.

8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify

constraints and trade –offs to be considered.

8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.

8.2.2.E.1: List and demonstrate the steps to an everyday task.

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

Students will understand that…

- The language of mathematics is communicated through specialized vocabulary and symbols used to represent and describe mathematical ideas, generalizations, and relationships.
- Representing mathematical ideas involves using a variety of representations such as graphs, numbers, algebra, words, and physical models to convey practical situations.

Essential Question(s): (What provocative questions will foster inquiry, understanding, and transfer of learning?)

What are the ways in which functions can be used to represent and solve problems involving quantities?

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	**E,T,A**	**Critical Thinking & Problem Solving**

	Environmental Literacy	E,T	Creativity and Innovation
	Health Literacy	E,T	Collaboration, Teamwork and Leadership
	Civic Literacy	E,T	Cross-Cultural and Interpersonal Communication
	Financial, Economic, Business and Entrepreneurial Literacy	E,T	Communication and Media Fluency
		E,T	Accountability, Productivity and Ethics

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

E	CRP1. Act as a responsible and contributing citizen and employee
E, T, A	CRP2. Apply appropriate academic and technical skills
E	CRP3. Attend to personal health and financial well-being
E, T, A	CRP4. Communicate clearly and effectively with reason
E	CRP5. Consider the environmental, social and economic impacts of decisions
E, T	CRP6. Demonstrate creativity and innovation
E	CRP7. Employ valid and reliable research strategies
E, T, A	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them
E, T	CRP9. Model integrity, ethical leadership and effective management
E	CRP10. Plan education and career paths aligned to personal goals
E, T	CRP11. Use technology to enhance productivity
E	CRP12. Work productively in teams while using cultural global competence

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
1. Key features of Functions 2. Transformations of Functions 3. Piecewise- Defined Functions 4. Arithmetic Sequences and Series 5. Solving Equations and inequalities by graphing 6. Linear Systems 7. Solving Linear Systems using Matrices	1. Write the domain and range of functions using set-builder and interval notations. 2. Identify key features of a graph of a function, including x-intercepts, y-intercepts, positive and negative intervals, and areas where the function is increasing or decreasing. 3. Calculate and interpret the average rate of change of a function over a specified interval. 4. Apply transformations to graph functions and write equations. 5. Graph and interpret piecewise-defined functions. 6. Interpret arithmetic sequences and series. 7. Use graphs and tables to approximate solutions to algebraic equations and inequalities. 8. Use a variety of tools to solve systems of linear equations and inequalities. 9. Solve Systems of Equations using Matrices
Assessment Evidence:	
Performance Tasks: *(Through what authentic performance tasks will students demonstrate the desired understandings? By what criteria will performances of understanding be judged?)* F.BF.A.1b A Sum of Functions F.BF.B.3 Exploring Sinusoidal Functions F.BF.B.3 Transforming the graph of a function	**Other Assessment Measures*:*** *(Through what other evidence (E.g. quizzes, tests, academic prompts, observations, homework, journals, etc.) will students demonstrate achievement of the desired results? How will students reflect upon and self- assess their learning?)* ******Attach all Benchmarks*** ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.***

F.IF.C.7c Graphs of Power Functions	
Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?	
Instructional Strategies and Activities *D*	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the envision Algebra 2 textbook by Pearson. If using another resource, please align content to what is identified in this plan.*** By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment. Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems. As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting. Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson. Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson.
Resources	
Pearson envision Algebra 2 Textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & EngageNY https://www.engageny.org/	
Suggested Time Frame:	CP: 25-30 days Adv: 22-25 days Honors: 20 days

D – Indicates differentiation at the Lesson Level

Content Area:	Algebra 2	Grade(s) 9-12
Unit Plan Title:	Unit 2 Quadratic Equations, Quadratic Functions, and Polynomial Functions	

Domain(s)

F.IF; A.SSE; A.APR; A.REI; & N-CN; A.SSE;A.CED; F.BF

Overview/Rationale

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

Equations & Inequalities plus Creating Equations: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Building Functions, Interpreting Functions and Linear & Exponential Models: Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Interpreting Functions : Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

The Real Number Systems: In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by imaginary numbers to form complex numbers.

Quantities: Quantities has a real application to real world problems. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in

modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Content Standard(s) (Established Goals)

College Prep:

F.BF.A.1: Write a function that describes a relationship between two quantities.*

F.BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.IF.B.4: For a functions that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **

F.IF.B.6: Calculate and interpret the average rate of change of a function(presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

F.IF.C.7: Graph a function expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

F.IF.C.8: Write a functions defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description) For example, given a graph of one quadratic function and an algebraic expression as another, say which has the larger maximum.

A.SSE.A.2: Use the structure of an expression to identify ways to rewrite it. *For example, see x^4-y^4 as $(x^2)^2-(y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$.*

A.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the Expression.

A.APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operation of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.B.2: Know and apply the Remainder Theorem: *For a polynomials p(x) and a number a, the reminder on division by x-a is p(a), so p(a)=0 if and only if (x-a) is a factor of p(x).*

A.APR.B.3: Identify zeros of polynomials when suitable factorization are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A.APR.C.4: Prove polynomials identify and use them to describe numerical relationships. *For example. The difference of two squares; the sum of difference of two cubes; the polynomials identity* $(x^2+y^2)^2+(x^2-y^2)^2+(2xy)^2$ *can be used to Pythagorean triples.*
A.APR.C.5: (+) Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
A.APR.D.6: Rewrite simple rational expressions in different forms; write a(x)b(x) in the form q(x)+r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degrees of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
A.REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the point of intersection between the line y+-3x and the circle* $x^2+y^2=3$.
A.REI.D.11: Explain why the x-coordinates of the equation y=f(x) and y=g(x) intersects are the solutions of the equation f(x)=g(x); find the solutions approximately. E.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f0x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
N.CN.C.8: (+) Extend polynomial identities to the complex numbers. For example, rewrite x^2+4 as (x+2i)(x-2i).
N.CN.C.9: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Advanced & Honors: The above standards plus:
A.REI.B.4: Solve quadratics equations by inspection (e.g., for $x^2= 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a\pm bi$ for real numbers a and b.
N.CN.A.1: Know there is a complex number i such that $i^2=-1$, and every complex number has the form $a+bi$ with a and b real.
N.CN.A.2: Use the relation $i^2=-1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N.CN.A.3: Find the conjugate of a complex number; use conjugate to find moduli and quotients of complex numbers.
N.CN.C.7: Solve quadratic equations with real coefficients that have complex solutions.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeating reasoning.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.

8.1.5.A.3: Use graphic organizers to organize information about problem or issue.

8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.

8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.

8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.

8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.

8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.

8.2.2.E.1: List and demonstrate the steps to an everyday task.

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

- The language of mathematics is communicated through specialized vocabulary and symbols used to represent and describe mathematical ideas, generalizations, and relationships.
- Representing mathematical ideas involves using a variety of representations such as graphs, numbers, algebra, words, and physical models to convey practical situations.
- Mathematical problem solvers apply a variety of strategies and methods to solve problem situations.

Essential Question(s): (What provocative questions will foster inquiry, understanding, and transfer of learning?)

1. How do you use quadratic functions to model situations and solve problems?
2. What can the rule for polynomial functions reveal about its graph, and what can the graphs of polynomial functions reveal about the solutions of polynomial equations?

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply. 21st Century Themes		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* 21st Century Skills	
	Global Awareness	**E, T, A**	**Critical Thinking & Problem Solving**
	Environmental Literacy	**E**	**Creativity and Innovation**
	Health Literacy	**E**	**Collaboration, Teamwork and Leadership**
	Civic Literacy	**E**	**Cross-Cultural and Interpersonal Communication**

	Financial, Economic, Business and Entrepreneurial Literacy	E	Communication and Media Fluency
		E	Accountability, Productivity and Ethics

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

E	**CRP1. Act as a responsible and contributing citizen and employee**
E, T	**CRP2. Apply appropriate academic and technical skills**
E	**CRP3. Attend to personal health and financial well-being**
E, T, A	**CRP4. Communicate clearly and effectively with reason**
E	**CRP5. Consider the environmental, social and economic impacts of decisions**
E, T	**CRP6. Demonstrate creativity and innovation**
E, T	**CRP7. Employ valid and reliable research strategies**
E, T, A	**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them**
E, T	**CRP9. Model integrity, ethical leadership and effective management**
E, T	**CRP10. Plan education and career paths aligned to personal goals**
E, T	**CRP11. Use technology to enhance productivity**
E, T	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
1. Vertex form, Standard form, Factor Form of a Quadratic Function	1. Create. Graph, and interpret quadratic functions.

2. Understand complex numbers and operations 3. Completing the Square 4. The Quadratic Formula 5. Graphing Polynomial Functions 6. Add, Subtract, Multiply, and Divide Polynomials 7. Polynomial Identities 8. Theorems about Roots of Polynomial Equations 9. Transformations of Polynomial Functions	2. Use Standard Form to create a quadratic function. 3. Add, subtract, and multiply complex numbers. 4. Use completing the square to transform a quadratic function. 5. Use the Quadratic Formula to solve quadratic equations that have complex solutions. 6. Graph, predict, and sketch polynomial functions. 7. Add, subtract, multiply, and divide polynomials 8. Prove polynomial identities 9. Extend polynomial theorems and identities to find real and complex number solutions
Assessment Evidence:	
Performance Tasks: Students demonstrate their understanding of the standards through multi-step higher order thinking problems. Teachers will incorporate at least two of the following questions/activities from different standards into their instruction from illustrativemathematics.org. These tasks will be used as additional support when covering the material in the unit. Questions similar to the problems should be incorporated into formal assessments. Students should receive feedback from the teacher on their performance on Performance Tasks. Teachers can also use the Common Core Performance task at the beginning of each chapter in the Algebra 1 Common Core textbook by Pearson. ***College Prep:*** A.SSE.A.2 A Cubic Identity A.APR.B.3 Graphing from Factors III A.REI.D.11 Ideal Gas Law	**Other Assessment Measures*:*** Quizzes, tests, academic prompts, observations, homework, benchmarks, journals, review games, & wipe board activities ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.***

F.IF.B.4, F.IF.C.7e Model air plane acrobatics F.IF.C.8b Carbon 14 dating in practice I F.IF.C.7c Graphs of Power Functions F.IF.C.7e Logistic Growth Model F.IF.C.9 Throwing Baseballs F.BF.B.3 Exploring Sinusoidal Functions F.BF.B.3 Transforming the graph of a function F.IF.C.8b Carbon 14 dating in practice I F.IF.C.7c Graphs of Power Functions F.IF.C.7e Logistic Growth Model F.IF.C.9 Throwing Baseballs F.BF.B.3 Exploring Sinusoidal Functions F.BF.B.3 Transforming the graph of a function ***Advanced & Honors:*** N.CN.A.1 Complex number patterns N.CN.A.2 Powers of a complex number N.CN.C.7, A.REI.B.4b Completing the square	
Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?	
Instructional Strategies and Activities	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the enVision Algebra 2 textbook by Pearson. If using another resource, please align content to what is identified in this plan.***

D	By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment. Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems. As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting. Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson. Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson.
Resources	
Pearson envision Algebra 2 Textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & EngageNY https://www.engageny.org/	
Suggested Time Frame:	CP: 43-52 days Adv: 38-43 days Honors: 38 days

Content Area:	Algebra 2	Grade(s): 9-12
Unit Plan Title:	Unit 3: Radical Functions, Rational Exponents, Radical Functions, Exponential & Logarithmic Functions	

Domain(s)

A.SSE; A.REI; A.CED; A.APR; N.RN; F.BF; F.IF; & F.LE

Overview/Rationale

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

Equations & Inequalities plus Creating Equations: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Building Functions, Interpreting Functions and Linear & Exponential Models: Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Interpreting Functions : Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

The Real Number Systems: In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by imaginary numbers to form complex numbers.

Quantities: Quantities has a real application to real world problems. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in

modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Content Standard(s) (Established Goals)

College Prep

A.SSE.A.2: Use the structure of an expression to identify ways to rewrite it. *For example, see x^4-y^4 as $(x^2)^2-(y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2-y^2)(x^2+y^2)$.*

A.SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

A.SSE.B.3.C: Use the properties of exponents to transform expressions for exponential functions. For the expressions 1.15^t can be written as $(1.15^{1/12})=1.012^{12t}$ to reveal the appropriate equivalent monthly interest rate if the annual rate is 15%.

A.SSE.B.4: Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

A.REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.A.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.REI.D.11: Explain why the x-coordinates of the equation y=f(x) and y=g(x) intersects are the solutions of the equation f(x)=g(x); find the solutions approximately. E.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f0x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

A.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V=IR to highlight resistance R.*

N.RN.A1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3=5(^{1/3})^3$ to hold, so $(5^{1/3})^3$ must equal 5.

N.RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

F.BF.A.1b: Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these function to the model.*

F.BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and

negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F.BF.A.1c: (+) Compare functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
F.IF.A.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0)=f91)=1, f(n+1)=f(n)+ F9n-1) for n≥1.
F.IF.B.4: For a functions that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **
F.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**
F.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
F.IF.C.7d: (+) Graph rational functions; identify zeros and asymptotes when suitable factorizations are available, showing end behavior.
F.IF.C.7e: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions*). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
F.LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (includes reading from a table).
F.LE.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*
A.APR.D.6: Rewrite simple rational expressions in different forms; write a(x)b(x) in the form q(x)+r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degrees of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
A.APR.D.7: (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Advanced and Honors

F.BF.B.4a: Solve an equation of the form f(x)=c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x)=2x^3$ or $f(x)=9x+1)/(x-1)$ for $x\neq1$.*

F.BF.B.4c: (+) Read values of an inverse function from a graph or table, given that the function has an inverse.

F.IF.C.7b: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.LE.A.4: Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct}=d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeating reasoning.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.

8.1.5.A.3: Use graphic organizers to organize information about problem or issue.

8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.

8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.

8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.

8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.

8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.

8.2.2.E.1: List and demonstrate the steps to an everyday task.

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

- The language of mathematics is communicated through specialized vocabulary and symbols used to represent and describe mathematical ideas, generalizations, and relationships.
- Representing mathematical ideas involves using a variety of representations such as graphs, numbers, algebra, words, and physical models to convey practical situations.
- Mathematical problem solvers apply a variety of strategies and methods to solve problem situations.

Essential Question(s): (What provocative questions will foster inquiry, understanding, and transfer of learning?)

1. How do you calculate with functions defined as quotients of polynomials, and what are the key features of their graphs?
2. How are rational exponents and radical equations used to solve real-world problems?
3. How do you use exponential and logarithms functions to model situations and solve problems?

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply. 21st Centur y Themes			*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* 21st Centur y Skills	
	Global Awareness		E,T,A	Critical Thinking & Problem Solving
	Environmental Literacy		E,T	Creativity and Innovation
	Health Literacy		E,T	Collaboration, Teamwork and Leadership
	Civic Literacy		E,T	Cross-Cultural and Interpersonal Communication
	Financial, Economic, Business and Entrepreneurial Literacy		E	Communication and Media Fluency
			E	Accountability, Productivity and Ethics

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

	E,T	CRP1. Act as a responsible and contributing citizen and employee
	E,T,A	CRP2. Apply appropriate academic and technical skills
	E	CRP3. Attend to personal health and financial well-being
	E,T,A	CRP4. Communicate clearly and effectively with reason
	E	CRP5. Consider the environmental, social and economic impacts of decisions
	E, T	CRP6. Demonstrate creativity and innovation
	E	CRP7. Employ valid and reliable research strategies
	E,T,A	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them

	E	**CRP9. Model integrity, ethical leadership and effective management**
	E	**CRP10. Plan education and career paths aligned to personal goals**
	E	**CRP11. Use technology to enhance productivity**
	E	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
1. Inverse Variation and the Reciprocal Function 2. Graphing Rational Functions 3. Add, Subtract, Multiply, and Divide Rational Expressions 4. Solve Rational Equations 5. Properties of exponents and radicals 6. Graph and Solve radical Equations 7. Function Operations 8. Inverse Relations and Functions 9. Key Features of Exponential Functions 10. Understand exponential models and Logarithms 11. Understand Logarithmic functions & properties 12. Exponential and Logarithmic Equations 13. Geometric Sequences & Series	1. Use inverse variations to write and graph reciprocal functions 2. Graph rational functions by identifying asymptotes and end behavior 3. Rewrite simple rational expressions in different forms using long division 4. Add, Subtract, Multiply, and Divide rational expression 5. Solve rational equations in one variable 6. Identify extraneous solutions to rational equations and give examples of how they arise 7. Find all real nth roots of a number 8. Evaluate expressions with rational exponents. 9. Use the properties of exponents and radicals to identify ways to rewrite radical expressions. 10. Interpret radical expressions 11. Graph radical functions. 12. Solve radical equations with one variable 13. Solve radical inequalities and apply the solutions within a real-world context.

	14. Combine functions by adding, subtracting, multiplying, and dividing and identify the domain result 15. Graph transformations of exponential function 16. Rewrite exponential functions to identify rates 17. Construct exponential models 18. Understand inverse relationships between exponents and logarithms 19. Graph logarithmic functions 20. Write equations that are inverses of exponential and logarithmic functions 21. Solve exponential and logarithmic equations 22. Construct a geometric sequence given a graph, table, or description of a relationship 23. Translate between geometric sequences written in recursive and explicit forms.
Assessment Evidence:	
Performance Tasks: Students demonstrate their understanding of the standards through multi-step higher order thinking problems. Teachers will incorporate at least two of the following questions/activities from different standards into their instruction from illustrativemathematics.org. These tasks will be used as additional support when covering the material in the unit. Questions similar to the problems should be incorporated into formal assessments. Students should receive feedback from the teacher on their performance on Performance Tasks. Teachers can also use the Common Core Performance task at the beginning of each chapter in the Algebra 1 Common Core textbook by Pearson.	**Other Assessment Measures*:*** Quizzes, tests, academic prompts, observations, homework, benchmarks, journals, review games, & wipe board activities ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.***

College Prep N.RN.A.1 Evaluating Exponential Expressions N.RN.A.2 Rational or Irrational? A.REI.A.2 Radical Equations A.REI.A.2, A.CED.A.1 An Extraneous Solution F.BF.A.1b A Sum of Functions F.IF.C.8b Carbon 14 dating in practice I F.IF.C.7c Graphs of Power Functions F.IF.C.7e Logistic Growth Model A.REI.D.11 Ideal Gas Law **Advanced and Honors** F.BF.B.4a Temperatures in degrees Fahrenheit and Celsius F.IF.B.4, F.IF.C.7e Model air plane acrobatics F.IF.C.9 Throwing Baseballs	

Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?

Instructional Strategies and Activities *D*	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Algebra 2 enVision textbook by Pearson. If using another resource, please align content to what is identified in this plan.*** By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment. Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems.

	As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting. Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson. Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson.
Resources	
Pearson envision Algebra 2 Textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & EngageNY https://www.engageny.org/	
Suggested Time Frame:	CP: 60-69 days Adv: 47-56 days Honors: 42-45 days

Content Area:	Algebra 2	Grade(s): 9-12
Unit Plan Title:	Unit 4: Probability, Data Analysis and Statistics	

Domain(s)

S-CP; S-IC; S-MD; & S-ID

Overview/Rationale

Statistics and Probability- Decisions or predictions are often based on data- numbers in content. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provided tools for describing variability in data and for making informed decisions that take it into account.

Content Standard(s) (Established Goals)

CP

S.IC.A.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.IC.B.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each

S.IC.B.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling

S.IC.B.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S.IC.B.6: Evaluate reports based on data.

Adv. & Honors

S.CP.A.1: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S.CP.A.2: Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.A.3: Understand the conditional probability of *A* given *B* as $P(A \text{ and } B)/P(B)$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

S.CP.A.4: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a*

random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
S.CP.A.5: Recognize and explain the NEW Concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
S.CP.B.6: Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model.
S.CP.B.7: Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
S.CP.B.8: (+) Apply the general Multiplication Rule in a uniform probability model, P9A and B)= P(A)P(BIA)= P(B)p(AIB), and intercept the answer in terms of the model.
S.CP.B.9: Use permutations and combinations to compute probabilities of compound events and solve problems.
S.IC.A.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
S.MD.B.6: (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
S.MD.B.7: (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
S.ID.A.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeating reasoning.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.

8.1.5.A.3: Use graphic organizers to organize information about problem or issue.

8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.

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are predictable?)

- The language of mathematics is communicated through specialized vocabulary and symbols used to represent and describe mathematical ideas, generalizations, and relationships.
- Representing mathematical ideas involves using a variety of representations such as graphs, numbers, algebra, words, and physical models to convey practical situations.

Essential Question(s) : (What provocative questions will foster inquiry, understanding, and transfer of learning?)

1. What is the difference between a permutation and a combination?
2. What is the difference between experimental and theoretical probability?
3. How are measures of central tendency different from standard deviation?

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply. 21st Centur y Themes			*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* 21st Centur y Skills	
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	E,T	CRP1. Act as a responsible and contributing citizen and employee
	E,T,A	CRP2. Apply appropriate academic and technical skills

E	**CRP3. Attend to personal health and financial well-being**
E,T	**CRP4. Communicate clearly and effectively with reason**
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E	**CRP11. Use technology to enhance productivity**
E	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
1. Basic terms of Probability	1. Use combinations and permutations accurately
2. Analyze data	2. Find experimental and theoretical probability of a events
3 Know the difference between a combination and permutation	3. Use tables to find the probability of an event
4. Know the difference between experimental and theoretical probability	4. Find the probability of multiple events

Assessment Evidence:

Performance Tasks: Students demonstrate their understanding of the standards through multi-step higher order thinking problems. Teachers will incorporate **at least one** of the following questions/activities from **each standard** into their instruction from illustrativemathematics.org. These tasks will be used as additional support when covering the material in the unit. Questions	**Other Assessment Measures*:*** Quizzes, tests, academic prompts, observations, homework, benchmarks, journals, review games, & wipe board activities ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned**

similar to the problems should be incorporated into formal assessments. Students should receive feedback from the teacher on their performance on Performance Tasks. Teachers can also use the Common Core Performance task at the beginning of each chapter in the Algebra 1 Common Core textbook by Pearson. S.ID.A.4 Do You Fit in This Car? S.IC.A.1School Advisory Panel S.IC.A.2 Sarah, the chimpanzee S.IC.B.3 Strict Parents S.IC.B.4 Margin of Error for Estimating a Population Mean S.CP.A.1 Describing Events S.CP.A.2 Cards and Independence S.CP.A.3 Lucky Envelopes S.CP.A.4 Two-Way Tables and Probability S.CP.A.5 Breakfast Before School S.CP.B.6 The Titanic 1 S.CP.B.7 The Addition Rule S.CP.B.7 Rain and Lightning	***into the department supervisor.***
Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?	
Instructional Strategies and Activities	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Algebra 2 enVision textbook by Pearson. If using another resource, please align content to what is identified in this plan.***

D	By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment. Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems. As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting. Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson. Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson.
Resources	
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Suggested Time Frame:	CP: 18-20 days Adv: 18-20 days Honors: 28-30 days

Curricular Map of High School Student Learning Standards for Mathematics

	Algebra	Geometry	Algebra II	Pre calculus
NUMBER AND QUANTITY (N)				
The Real Number System (RN)				
N-RN.A.1.Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	X		X	X
N-RN.A.2.Rewrite expressions involving radicals and rational exponents using the properties of exponents.	X		X	X
N-RN.B.3.Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	X			X
Quantities (Q)				
N-Q.A.1.Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	X	X		X
N-Q.A.2.Define appropriate quantities for the purpose of descriptive modeling.	X	X	X	X
N-Q.A.3.Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	X			X
The Complex Number System (CN)				
N-CN.A.1.Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real.			X	
N-CN.A.2.Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.			X	
N-CN.A.3.(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.			X	X

	Algebra	Geometry	Algebra II	Pre calculus
N-CN.B.4.(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.				X
N-CN.B.5.(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.				X
N-CN.B.6.(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				X
N-CN.C.7.Solve quadratic equations with real coefficients that have complex solutions.			X	
N-CN.C.8.(+) Extend polynomial identities to the complex numbers.			X	X
N-CN.C.9.(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.			X	X
Vector and Matrix Quantities (VM)				
N-VM.A.1.(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v, 14 114,** v).		X		X
N-VM.A.2.(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				X
N-VM.A.3.(+) Solve problems involving velocity and other quantities that can be represented by vectors.				X

	Algebra	Geometry	Algebra II	Pre calculus
N-VM.B.4. (+) Add and subtract vectors. d. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. e. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. f. Understand vector subtraction v— was v + (—w), where —w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.		X		
N-VM.B.5. (+) Multiply a vector by a scalar. c. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vv) = (mix, cv_i). d. Compute the magnitude of a scalar multiple cv using licv11= iciv. Compute the direction of cv knowing that when Iclv # 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).				X
N-VM.C.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.				X
N-VM.C.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.				X
N-VM.C.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.				X
N-VM.C.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.				X
N-VM.C.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.				X

	Algebra	Geometry	Algebra II	Pre calculus
N-VM.C.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.				X
N-VM.C.12. (+) Work with 2 x 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.				X
Algebra (A)				
Seeing Structure in Expressions (SSE)				
A-SSE.A.1. Interpret expressions that represent a quantity in terms of its context.* c. Interpret parts of an expression, such as terms, factors, and coefficients. d. Interpret complicated expressions by viewing one or more of their parts as a single entity.	X	X	X	X
A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it.	X		X	X
A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* d. Factor a quadratic expression to reveal the zeros of the function it defines. e. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. f. Use the properties of exponents to transform expressions for exponential functions.	X	X	X	X
A-SSE.B.4. Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.			X	X
Arithmetic with Polynomials and Rational Expressions (APR)				
A-APR.A.1.Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	X		X	
A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	X		X	X

	Algebra	Geometry	Algebra II	Pre calculus
A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	X		X	X
A-APR.C.4. Prove polynomial identities and use them to describe numerical relationships.			X	
A-APR.C.5. (+) Know and apply the Binomial Theorem for the expansion of *(x* + y)" in powers of *x* and y for a positive integer *n,* where *x* and y are any numbers, with coefficients determined for example by Pascal's Triangle.			X	
A-APR.D.6.Rewrite simple rational expressions in different forms; write $a^{(}x^{)}/_{b(x)}$ in the form *q(x)* + $^{trx)}$/ *b(x),* where a(x), *b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x),* using inspection, long division, or, for the more complicated examples, a computer algebra system.	X		X	
A-APR.D.7.(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	X		X	
Creating Equations (CED)				
A-CED.A.1.Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	X			
A-CED.A.2.Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	X	X		
A-CED.A.3.Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	X		X	
A-CED.A.4.Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	X			X

	Algebra	Geometry	Algebra II	Pre calculus
Reasoning with Equations and Inequalities (REI)				
A-REI.A.1.Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**X**		**X**	**X**
A-REI.A.2.Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**X**		**X**	**X**
A-REI.B.3.Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**X**			**X**
A-REI.B.4. Solve quadratic equations in one variable. c. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. d. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b.*	**X**		**X**	**X**
A-REI.C.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**X**			
A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	**X**		**X**	
A-REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.			**X**	
A-REI.C.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.				**X**
A-REI.C.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).				**X**

	Algebra	Geometry	Algebra II	Pre calculus
A-REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	X			
A-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y= *f(x)* and y = *g(x)* intersect are the solutions of the equation *f(x) = g(x);* find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	X	X		X
A-REI.D.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	X			
FUNCTIONS (F)				
Interpreting Functions (IF)				
F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x.* The graph of *f* is the graph of the equation y= *f(x).*	X			X
F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	X			X
F-IF.A.3.Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	X		X	
F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	X			X
F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it descrbes.*	X			X

	Algebra	Geometry	Algebra II	Pre calculus Math Topics
F-IF.B.6.Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	X		X	X
F-IF.C.7.Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* f. Graph linear and quadratic functions and show intercepts, maxima, and minima. g. Graph square root, cube root, and piecewise-defined functions, including step *functions* and absolute value functions. h. Graph polynomial functions, identifying zeros when suitable factorizations are *available,* and showing end behavior. *i.* (+) Graph rational functions, identifying zeros and asymptotes when suitable *factorizations* are available, and showing end behavior. j. Graph exponential and logarithmic functions, showing intercepts and end *behavior,* and trigonometric functions, showing period, midline, and amplitude.	X		X	X
F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. c. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. d. Use the properties of exponents to interpret expressions for exponential functions.	X		X	
F-IF.C.9.Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	X		X	
Building Functions (BF)				
F-BF.A.1.Write a function that describes a relationship between two quantities.* d. Determine an explicit expression, a recursive process, or steps for calculation from a context. e. Combine standard function types using arithmetic operations. f. (+) Compose functions.	X		X	X
F-BF.A.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	X		X	X

	Algebra	Geometry	Algebra II	Pre calculus
F-BF.B.3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	X		X	X
F-BF.B.4. Find inverse functions. e. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. f. (+) Verify by composition that one function is the inverse of another. g. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. h. (+) Produce an invertible function from a non-invertible function by restricting the domain.			X	X
F-BF.B.5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving exponents and logarithms.			X	X

	Algebra	Geometry	Algebra II	Pre calculus
Linear, Quadratic and Exponential Models (LE)				
F-LE.A.1.Distinguish between situations that can be modeled with linear functions and with exponential functions. d. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. e. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. f. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	X			X
F-LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	X		X	
F-LE.A.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	X			
F-LE.A.4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e;* evaluate the logarithm using technology.			X	X
F-LE.B.5.Interpret the parameters in a linear or exponential function in terms of a context.	X			
Trigonometric Functions (TF)				
F-TF.A.1.Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.			X	X
F-TF.A.2.Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.			X	X
F-TF.A.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for Tr13, Tr/4 and Tr/6, and use the unit circle to express the values of sine, cosines, and tangent for *x,* IF + *x,* and 2¯rr — *x* in terms of their values for *x,* where *x* is anyreal number.				X

	Algebra	Geometry	Algebra H	Pre calculus
F-TF.A.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.				X
F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*			X	X
F-TF.B.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.				X
F-TF.B.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*				X
F-TF.C.8. Prove the Pythagorean identity $\sin^2(0) + \cos^2(0) = 1$ and use it to find sin(0), cos(e), or tan(0) given sin(0), cos(0), or tan(0) and the quadrant of the angle.			X	X
F-TF.C.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.				X
GEOMETRY (G)				
Congruence (CO)				
G.CO.A.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.		X		
G-CO.A.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).		X		
G-CO.A.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.		X		
G-CO.A.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		X		
G-CO.A.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.		X		
G-CO.B.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.		X		
G-CO.B.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.		X		

	Algebra	Geometry	Algebra II	Pre calculus
G-CO.B.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.		X		
G-CO.C.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*		**X**		**X**
G-CO.C.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*		**X**		**X**
G-CO.C.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*		**X**		
G-CO.D.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*		**X**		
G-CO.D.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.		**X**		
Similarity, Right Triangles, and Trigonometry (SRT)				
G-SRT.A.1. Verify experimentally the properties of dilations given by a center and a scale factor: c. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. d. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.		**X**		
G-SRT.A.2.Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.		X		
G-SRT.A.3.Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	**X**	**X**		
G-SRT.B.4.Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*		**X**		

	Algebra	Geometry	Algebra II	Pre calculus
G-SRT.B.5.Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.		X		X
G-SRT.C.6.Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		X	X	X
G-SRT.C.7.Explain and use the relationship between the sine and cosine of complementary angles.		X		X
G-SRT.C.8.Use trigonometric ratios and Pythagorean Theorem to solve right triangles in applied problems.*	X	X		X
G-SRT.D.9.(+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.		X		X
G-SRT.D.10.(+) Prove the Laws of Sines and Cosines and use them to solve problems.		X		X
G-SRT.D.11.(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).		X		X
Circles (C)				
G-C.A.1.Prove that all circles are similar.		X		
G-C.A.2.Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*		*X*		
G-C.A.3.Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.		X		
G-C.A.4.(+) Construct a tangent line from a point outside a given circle to the circle.		X		
G-C.B.5.Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.		X		
Expressing Geometric Properties with Equations (GPE)				
G-GPE.A.1.Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		X		X
G-GPE.A.2.Derive the equation of a parabola given a focus and directrix.			X	X
G-GPE.A.3.(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	X			X

	Algebra	Geometry	Algebra II	Pre calculus
G-GPE.B.4.Use coordinates to prove simple geometric theorems algebraically.		X	X	
G-GPE.B.5.Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	X	X		
G-GPE.B.6.Find the point on a directed line segment between two given points that partitions the segment in a given ratio.		X		
G-GPE.B.7.Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*				
Geometric Measurement and Dimension (GMD)				
G-GMD.A.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*		X		X
G-GMD.A.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.		X		X
G-GMD.A.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*		X		X
G-GMD.B.4. Identify the shape of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.		X		
Modeling with Geometry (MG)				
G-MG.A.1.Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*		X		X
G-MG.A.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*				
G-MG.A.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*		X		
Statistics and Probability (S)				
Interpreting Categorical and Quantitative Data (ID)				
S-ID.A.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	X			

	Algebra	Geometry	Algebra II	Pre calculus
S-ID.A.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	X			X
S-ID.A.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	X			X
S-ID.A.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	X			X
S-ID.B.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	X			X
S-ID.B.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. d. Fit a function to the data; use functions fitted to data (including with the use of technology) to solve problems in the context of the data. Use given functions or chooses a function suggested by the context. Emphasize linear, quadratic, and exponential models. e. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology f. Fit a linear function for a scatter plot that suggests a linear association.	X			X
S-ID.C.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	X			
S-I D.C.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	X			
S-ID.C.9. Distinguish between correlation and causation.	X			
Making Inferences and Justifying Conclusions (IC)				
S-IC.A.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.			X	X

	Algebra	Geometry	Algebra II	Pre calculus
S-IC.A.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.			X	X
S-IC.B.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.			X	
S-IC.B.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.			X	X
S-IC.B.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.			X	
S-IC.B.6. Evaluate reports based on data.			X	
Conditional Probability and the Rules of Probability (CP)				
S-CP.A.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").			X	X
S-CP.A,2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.			X	X
S-CP.A.3. Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B),* and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A,* and the conditional probability of *B* given *A* is the same as the probability of *B.*			X	X
S-CP.A.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.			X	
S-CP.A.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.			X	X
S-CP.B.6. Find the conditional probability of *A* given *B* as the fraction of B's outcomes that also belong to *A,* and interpret the answer in terms of the model.			X	X
S-CP.B.7. Apply the Addition Rule, P(A or B) = P(A) + P(B) — P(A and B), and interpret the answer in terms of the model.			X	X
S-CP.B.8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(BIA) = P(B)P(AIB), and interpret the answer in terms of the model.				X

	Algebra	Geometry	Algebra II	Pre-calculus
S-CP.B.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.				X
Using Probability to Make Decisions (MD)				
S-MD.A.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.				
S-MD.A.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.				
S-MD.A.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.				
S-MD.A.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.				
S-MD.B.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. c. Find the expected payoff for a game of chance. d. Evaluate and compare strategies on the basis of expected values.				
S-MD.B.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).				
S-MD.B.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).				

APPENDIX A: Course Syllabus

Color Coding: All Levels Advanced/Honors Honors only

UNIT 1

Chapter 1

1-1 1-2 1-3 1-4 1-5 1-6 1-7

UNIT 2

Chapter 2

2-1 2-2 2-3 2-4 2-5 2-6 2-7

Chapter 3

3-1 3-2 3-3 3-4 3-5 3-6 3-7

UNIT 3

Chapter 4

4-1 4-2 4-3 4-4 4-5 4-6

Chapter 5

5-1 5-2 5-3 5-4 5-5 5-7

Chapter 6

6-1 6-2 6-3 6-4 6-5 6-6 6-7

UNIT 4

Chapter 11

11-1 11-2 11-3 11-4 11-5 11-6

Chapter 12

12-1 12-2 12-3 12-4 12-5 12-6

APPENDIX B: Webb's Depth-of-Knowledge (DOK) Levels for Mathematics created by Norman Webb

According to Norman L. Webb, Wisconsin Center for Educational Research ("Depth-of-Knowledge Levels for Four Content Areas," March 22, 2002), "interpreting and assigning depth-of-knowledge levels to both objectives within standards and assessment items is an essential requirement of alignment analysis. Four levels of depth-of-knowledge are used for this analysis." Norman Webb's "Depth-of-Knowledge Levels for four Content Areas" include: Language Arts (reading, Writing), Mathematics, Science, and Social Studies.

A general definition for each of the four (Webb) Depth-of-Knowledge levels is followed by Table 1, which provide further specifications and examples for each of the DOK levels. Webb recommends that large scale, on-demand assessments in mathematics should only assess Depth of Knowledge Levels 1, 2, and 3. Depth of Knowledge at Level 4 in mathematics should be reserved for local assessment.

Level 1 (Recall) includes the recall of information such as fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify", "recall", "recognize", "use", and "measure". Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify", "organize", "estimate", "make observations", "collect and display data", and "compare data". These actions imply more than one step. For example, to compare data requires first identifying characteristics of the object or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain", "describe", or "interpret" could be classified at different levels depending on the object of the action. For example, if an item requires students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills

because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level of other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be complex. Students should be required to make several connections-relate ideas *within* the content area or *among* content areas-and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

Math Descriptors- Combined Webb Depth of Knowledge Levels of Mathematics (Webb, 2002), NAEP 2002 Mathematics Levels of Complexity, and Other Descriptors related to NECAP GLEs.

LEVEL 1 RECALL	LEVEL 2 SKILLS/CONCEPTS	LEVEL 3 STRATEGIC THINKING	LEVEL 4 EXTENDED THINKING
Examples represent, but do not constitute all Level 1 mathematics performances: Recall or recognize a fact, definitions, or terms Apply a well known algorithm Apply a formula Determine the area or perimeter of rectangles or triangles given drawing and labels Identify a plan or three dimensional figure Measure a length Perform a specified or routine procedure Evaluate an expression	**Examples represent, but do not constitute all Level 2 mathematics performances:** Classify plane and three dimensional figures Interpret information from a simple graph Use models to represent mathematical concept Solve a **routine** problem requiring multiple steps, or the application of multiple concepts Compare figures or statements Provide justifications for steps in a solution process Extend a pattern	**Examples represent, but do not constitute all Level 3 mathematics performances:** Interpret information from a complex graph Explain thinking when more than one response is possible Make and/or justify conjectures Develop logical arguments for a concept Use concepts to solve problems Perform procedure with multiple steps and multiple decision points Generalize a pattern Describe, compare and contrast solution methods	**Examples represent, but do not constitute all Level 4 mathematics performances:** Relate mathematical concepts to other content areas Relate mathematical concepts to real-world applications in new situation Apply a mathematical model to illuminate a problem, situation Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results Design a mathematical model to inform and solve a practical or abstract situation ***NOTE:*** **Level 4 requires applying**

Solve a one-step word problem Retrieve information from a table or graph Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percents), or within and between customary and metric measures Locate numbers on a number line, or points on a coordinate grid Solves linear equations Represent math relationships in words, pictures, or symbols	Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps Translate between tables, graphs, words and symbolic notation Select a procedure according to criteria and perform it	Formulate a mathematical model for a complex situation Provide mathematical justifications Solve a multiple step problem, supported with a mathematical explanation that justifies the answer Formulate an original problem, given a situation	**one approach among many to solve problems. Involves complex restructuring of data, establishing and evaluating criteria to solve problems.**

GEOMETRY CURRICULUM

Middle Township Public Schools
216 S. Main Street
Cape May Court House, NJ 08210

Born: August 2022

Revised: October 2023

Content Area:	High School Mathematics - Geometry	Grade(s) 9-12
Unit Plan Title:	UNIT 1 - Foundations of Geometry	

Domain(s)

G-CO Congruence
G-GPE Expressing Geometric Properties with Equations
G-MG Modeling with Geometry

Overview/Rationale

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

Content Standard(s)

All levels:

G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance around a circular arc.
G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
G-CO.C.9 Prove theorems about lines and angles.
G-MG.A.3 Apply geometric methods to solve design problems
G-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems
G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects

Advanced/Honors:

G-CO.D.12 Make formal geometric constructions with a variety of tools and methods

Math Practice Standards

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP4 Model with mathematics.
MP5 Use appropriate tools strategically.
MP6 Attend to precision.
MP7 Look for and make use of structure.

Integration of Technology

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

Interdisciplinary Standards

NJSLSA.R8 Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.

RST.9-10.5 Analyze the relationships among concepts in a text, including relationships among key terms.

Enduring Understandings:

- Points, lines, and planes are the foundations of geometry.
- Points, lines, and planes form the foundational properties of geometry.
- The measures of geometric figures can be calculated and analyzed using a variety of strategies, tools, and technologies.
- The structure of the coordinate system can be used to calculate measures contained in geometric figures.
- The Distance Formula is an application of the Pythagorean Theorem
- Proof is a justification that is logically valid and based on definitions, postulates, and theorems
- Valid inductive and deductive reasoning are used to develop and prove conjectures.
- Logical arguments consist of a set of premises or hypotheses and a conclusion
- Linear relationships can be defined on the coordinate plane
- Two intersecting lines form angles with specific relationships.
- Parallel lines cut by a transversal form angles with specific relationships.
- The slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals of each other

Essential Question(s):

All Levels:

- How are the properties of segments and angles used to determine their measures?
- How are the midpoint and length of a segment on the coordinate plane determined?
- How is deductive reasoning used to prove a theorem?

Advanced/Honors:

- How are a straightedge and compass used to make basic constructions?
- How is inductive reasoning used to recognize mathematical relationships?
- How do if-then statements describe mathematical relationships?

Honors Only:

- How is deductive reasoning different from inductive reasoning?

Career Education:

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.

9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions.

Student Learning Goals/Objectives:	
Students will know…. Key vocabulary <u>All levels</u> ● Angle bisector ● Proof ● Perpendicular bisector ● Postulate ● Theorem <u>Advanced/Honors</u> ● Inductive reasoning ● Conjecture ● Counterexample ● Conditional ● Truth table ● Truth value ● Contrapositive ● Converse ● Inverse ● Bi-conditional ● Negati on <u>Honors Only</u> ● Construction ● Deductive reasoning ● Law of detachment ● Law of syllogism	***Students will be able to (do)…*** ● Experiment with transformations in the plane ● Prove geometric theorems ● Make geometric constructions ● Use coordinates to prove simple geometric theorems algebraically ● Apply geometric concepts in modeling situations

Assessment Evidence:	
Alternative Assessments: Teachers will expose students to higher-order thinking problems that are similar to the NJSLA/NJGPA. These tasks will be used as additional support when covering the material in the unit. Students should receive feedback from the teacher on their performance on Performance Tasks. **Domain: G-CO** Congruence G.CO.A.1 G.CO.C.9 G.CO.C.10 G.CO.C.11 G.CO.D.12 **Domain: G-GPE** Expressing Geometric Properties with Equations G.GPE.5 G.GPE.6 **Domain: G-MG** Modeling with Geometry G.MG.A.1 G.MG.A.3	**Assessments (Diagnostic, Formative, Summative, Benchmark, Alternative)** ● Diagnostic: Course Readiness Assessments, Topic Readiness Assessments ● Formative: Independent problems during lessons, Lesson Quizzes ● Summative: Topic Assessments, Performance Assessments, Benchmark and Mid-Course Assessments, End of Course Assessments ● Alternative- one on one student conferencing and verbal understanding of concepts

Teaching and Learning Actions	
	Note: See attached list of lessons that should be taught throughout this unit. If using another resource, please align content to what is identified in this plan.
Instructional Strategies and Activities Review Games (e.g. Jeopardy)	By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment.
Guided Notes	Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems.
Higher-order Thinking Problems	As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting.
Activators	Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson.
Closure	Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson

Accommodations

- Special Education Students
    - Ongoing Interventions, strategic Interventions and Intensive Interventions that are teacher and/or technologically driven within the curriculum and targeted to standards in need of support.
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge.
    - Extended time for revisions or opportunity to identify and develop areas of personal interest
- English Language Learners
    - Bilingual language glossary for mathematical literacy and vocabulary prior to and during each lesson.
    - English Language Learner support embedded into each lesson for use during the instructional part of the lesson (rephrasing, models, photos,etc.)
    - Targeted support strategies that focus on key areas of language development: listening, speaking, reading, and writing for students at different proficiency levels.
    - Technology supported assignments aligned with standards in students native language.
- Students at Risk of School Failure
    - Teacher led reteach to build understanding which encompass guided reteaching activities for teachers to utilize when students need a better understanding of lesson topics.
    - Readiness assessment individualized study plan generated based on a students' score on the topic readiness assessment. Allows students to gain additional support in standards that should have been mastered in a previous grade.
    - Adaptive practice and homework utilizing recommended resources
    - Encourage students to make transformations - use a common task or item in a different way
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge.
    - Extended time for revisions or opportunity to identify and develop areas of personal interest
- 504 Students
    - Video tutorials for each lesson with units for students to access as needed.
    - Mathematical literacy and vocabulary activities that provide scaffolded support for students to build vocabulary
    - Encourage creative expression and thinking by allowing students to choose how to approach a problem or assignment.
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge.

- Extended time for revisions or opportunity to identify and develop areas of personal interest
- Gifted & Talented Students
  - Enrichment activities for teachers to utilize when learners are advanced within the given standard/topic.
  - Encourage students to explore concepts in depth and encourage independent studies or investigations.
  - Modeling or independent student led research

Core Instructional Materials and Supplemental Materials	
Savaas envision textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & EngageNY https://www.engageny.org/	
Suggested Time Frame:	College Prep: 32-34 days, Advanced: 27-29 days, Honors: 22-24 days

Content Area:	**High School Mathematics - Geometry**	**Grade(s) 9-12**
Unit Plan Title:	UNIT 2 - Transformations, Congruence, and Similarity	

Domain(s)

G-SRT Similarity, Right Triangles, and Trigonometry
G-CO Congruence
G-C Circles

Overview/Rationale

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combination of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally).
Similarity transformation define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

Content Standard(s)

All levels:

G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-CO.C.10 Prove theorems about triangles.

G-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems

G-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not

G-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draws the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.

G-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent

G-CO.B.8 Explain how the criteria for triangle congruence (ASA,SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

G-SRT.A.1 Verify experimentally the properties of dilation given by a center and a scale factor: a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged; the dilation of a line segment is longer or shorter in the ratio given by a scale factor

G-SRT.A.2 Given two figures use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pair of angles and the proportionality of a corresponding pairs of sides.

G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

G-SRT.B.4 Prove theorems about triangles

G-C.A.1 Prove that all circles are similar

Honors only: G-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself

Math Practice Standards
MP1 Make sense of problems and persevere in solving them. **MP2** Reason abstractly and quantitatively. **MP3** Construct viable arguments and critique the reasoning of others. **MP4** Model with mathematics. **MP6** Attend to precision. **MP7** Look for and make use of structure.
Integration of Technology
8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change. **8.1.12.DA.5:** Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

Interdisciplinary Standards
RST.9-10.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text. **RST.9-10.5** Analyze the relationships among concepts in a text, including relationships among key terms.

Enduring Understandings:

- Relationships that exist between the angles and segments of triangles can be proven.
- Similar and congruent geometric figures have proportional attributes
- A constant ratio exists between corresponding lengths of sides of similar figures.
- Geometric transformations are functional relationships.

Essential Question(s):

All levels

- How are properties of reflection used to transform a figure?
- What are the properties of a translation?
- What are the properties that identify a rotation?
- How can rigid motions be classified?
- What is the relationship between rigid motions and congruence?
- How are the side lengths and angle measures related in isosceles triangles and in equilateral triangles?
- How are SAS and SSS used to show that two triangles are congruent?
- How are ASA and AAS used to show that triangles are congruent?
- What minimum criteria are needed to show that right triangles are congruent?

Honors Only

- How can you tell whether a figure is symmetric?
- Which theorems can be used to prove that two overlapping triangles are congruent?

Career Education:

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions.

Student Learning Goals/Objectives:	
Students will know… Key vocabulary ● Composition of rigid motions ● Glide reflection ● Point symmetry ● Reflectional symmetry ● Rigid motion ● Rotational symmetry ● symmetry	***Students will be able to (do):*** ● Experiment with transformations in the plane ● Understand congruence in terms of rigid motions ● Prove geometric theorems ● Prove theorems involving similarity ● Understand similarity in terms of similarity transformations ● Understand and apply theorems about circles ● Use coordinates to prove simple geometric theorems algebraically

Assessment Evidence:	
Alternative Assessments: Teachers will expose students to higher-order thinking problems that are similar to the NJSLA/NJGPA. These tasks will be used as additional support when covering the material in the unit. Students should receive feedback from the teacher on their performance on Performance Tasks. **Domain: G-SRT** Similarity, Right Triangles, and Trigonometry G-SRT.A.1 G-SRT.A.2 G-SRT.A.3 G-SRT.B.4 G-SRT.B.5	**Assessments (Diagnostic, Formative, Summative, Benchmark, Alternative)** ● Diagnostic: Course Readiness Assessments, Topic Readiness Assessments ● Formative: Independent problems during lessons, Lesson Quizzes ● Summative: Topic Assessments, Performance Assessments, Benchmark and Mid-Course Assessments, End of Course Assessments ● Alternative- one on one student conferencing and verbal understanding of concepts

Domain: G-CO Congruence G.CO.A.2 G.CO.A.3 G.CO.A.4 G.CO.A.5 G.CO.B.6 G.CO.B.7 G.CO.B.8 **Domain: G-C** Circles G-C.A.1 G-C.A.3	

Teaching and Learning Actions	
	Note: See attached list of lessons that should be taught throughout this unit. If using another resource, please align content to what is identified in this plan.
Instructional Strategies and Activities Review Games (e.g. Jeopardy) Guided Notes	By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment. Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems.

Higher-order Thinking Problems	As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting.
Activators	Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson.
Closure	Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson

Accommodations

- Special Education Students
    - Ongoing Interventions, strategic Interventions and Intensive Interventions that are teacher and/or technologically driven within the curriculum and targeted to standards in need of support.
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge.
    - Extended time for revisions or opportunity to identify and develop areas of personal interest
- English Language Learners
    - Bilingual language glossary for mathematical literacy and vocabulary prior to and during each lesson.
    - English Language Learner support embedded into each lesson for use during the instructional part of the lesson (rephrasing, models, photos,etc.)
    - Targeted support strategies that focus on key areas of language development: listening, speaking, reading, and writing for students at different proficiency levels.
    - Technology supported assignments aligned with standards in students native language.
- Students at Risk of School Failure
    - Teacher led reteach to build understanding which encompass guided reteaching activities for teachers to utilize when students need a better understanding of lesson topics.
    - Readiness assessment individualized study plan generated based on a students' score on the topic readiness assessment. Allows students to gain additional support in standards that should have been mastered in a previous grade.
    - Adaptive practice and homework utilizing recommended resources
    - Encourage students to make transformations - use a common task or item in a different way
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge.
    - Extended time for revisions or opportunity to identify and develop areas of personal interest
- 504 Students
    - Video tutorials for each lesson with units for students to access as needed.
    - Mathematical literacy and vocabulary activities that provide scaffolded support for students to build vocabulary
    - Encourage creative expression and thinking by allowing students to choose how to approach a problem or assignment.

- Gifted & Talented Students
    - Enrichment activities for teachers to utilize when learners are advanced within the given standard/topic.
    - Encourage students to explore concepts in depth and encourage independent studies or investigations.
    - Modeling or independent student led research

Resources	
Savaas envision textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & EngageNY https://www.engageny.org/	
Suggested Time Frame:	College Prep: 54-56 days, Advanced: 41-43 days Honors: 28-30 days

Content Area:	High School Mathematics - Geometry	Grade(s) 9-12
Unit Plan Title:	UNIT 3 - Right Triangles, Area, and Volume	

Domain(s)

All Levels:

G-SRT Similarity, Right Triangles, and Trigonometry **G-GMD** Geometric Measurement and Dimension **G-MG** Modeling with Geometry

Overview/Rationale

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Law of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle.

Content Standard(s)

All levels:

G-SRT.B.4 Prove theorems about triangles

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems

G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles

G-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone

G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects

G-MG.A.2 Apply concepts of density based on area and volume in modeling situations

Honors Only: G-SRT.D.10 Prove the Law of Sines and Cosines and use them to solve problems

G-SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles

G-GMD.B.4 Use coordinates to prove simple geometric theorems algebraically

G-GMD.A.2 Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures

Math Practice Standards

MP1 Make sense of problems and persevere in solving them.
MP3 Construct viable arguments and critique the reasoning of others.
MP4 Model with mathematics.
MP5 Use appropriate tools strategically.
MP6 Attend to precision.
MP7 Look for and make use of structure.
MP8 Look for and express regularity in repeated reasoning.

Integration of Technology

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

Interdisciplinary Standard(s)

RST.9-10.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.5 Analyze the relationships among concepts in a text, including relationships among key terms.

Enduring Understandings:

- The special properties of right triangles can be used to solve real world problems.
- Problems involving two and three-dimensional measurements can be solved using geometric relationships and trigonometry.
- When the linear dimensions of an object change, the volume and surface area change in a predictable way.
- The properties of three-dimensional objects are related to those of two-dimensional objects.

Essential Question(s):

All levels

- How are similarity in right triangles and the Pythagorean Theorem related?
- How do trigonometric ratios relate angle measures to side lengths of right triangles?
- How can trigonometry be used to solve real-world and mathematical problems?
- How does the volume of a prism or cylinder relate to a cross section parallel to its base?
- How are the formulas for volume of a pyramid and volume of a cone alike?
- How does the volume of a sphere relate to the volumes of other solids?

Honors Only

- How can the Law of Sines be used to determine side lengths and angle measures in acute and obtuse triangles?
- How can the Law of Cosines be used to determine side lengths and angle measures in acute and obtuse triangles?
- How are space figures and polygons related?

Career Education:

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions.

Student Learning Goals/Objectives:

Students will know….	***Students will be able to (do):***
Key vocabulary <u>All levels</u> ● Angle of depression ● Angle of elevation ● Cosine ● Pythagorean triple	All levels ● Prove theorems involving similarity ● Define trigonometric ratios and solve problems involving right triangles ● Explain volume formulas as use them to solve problems ● Apply geometric concepts in modeling situations Honors only

• Sine • Tangent • Trigonometric ratios • Hemisphere • Oblique cylinder • Oblique prism Honors Only • Law of Cosines • Law of Sines • Cavalier's Principle	• Apply trigonometry to general triangles • Visualize relationships between two-dimensional and three-dimensional objects

Assessment Evidence:	
Alternative Assessments: Teachers will expose students to higher-order thinking problems that are similar to the NJSLA/NJGPA. These tasks will be used as additional support when covering the material in the unit. Students should receive feedback from the teacher on their performance on Performance Tasks. **Domain: G-SRT** Similarity, Right Triangles, and Trigonometry G-SRT.B.4 G-SRT.C.6 G-SRT.C.7 G-SRT.C.8 G-SRT.D.9 G-SRT.D.10 G-SRT.D.11	**Assessments (Diagnostic, Formative, Summative, Benchmark, Alternative)** • Diagnostic: Course Readiness Assessments, Topic Readiness Assessments • Formative: Independent problems during lessons, Lesson Quizzes • Summative: Topic Assessments, Performance Assessments, Benchmark and Mid-Course Assessments, End of Course Assessments • Alternative- one on one student conferencing and verbal understanding of concepts

Domain: G-GMD Geometric Measurement and Dimension G-GMD.A.1 G-GMD.A.2 G-GMD.A.3 G-GMD.B.4 **Domain: G-MG** Modeling with Geometry G-MG.A.1 G-MG.A.2	

Teaching and Learning Actions	
	Note: See attached list of lessons that should be taught throughout this unit. If using another resource, please align content to what is identified in this plan.
Instructional Strategies and Activities Review Games (e.g. Jeopardy)	By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment.
Guided Notes	Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems.
Higher-order Thinking Problems	As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting.
Activators	

Closure	Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson. Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson

Accommodations

- Special Education Students
    - Ongoing Interventions, strategic Interventions and Intensive Interventions that are teacher and/or technologically driven within the curriculum and targeted to standards in need of support.
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge.
    - Extended time for revisions or opportunity to identify and develop areas of personal interest
- English Language Learners
    - Bilingual language glossary for mathematical literacy and vocabulary prior to and during each lesson.
    - English Language Learner support embedded into each lesson for use during the instructional part of the lesson (rephrasing, models, photos,etc.)
    - Targeted support strategies that focus on key areas of language development: listening, speaking, reading, and writing for students at different proficiency levels.
    - Technology supported assignments aligned with standards in students native language.
- Students at Risk of School Failure
    - Teacher led reteach to build understanding which encompass guided reteaching activities for teachers to utilize when students need a better understanding of lesson topics.
    - Readiness assessment individualized study plan generated based on a students' score on the topic readiness assessment. Allows students to gain additional support in standards that should have been mastered in a previous grade.
    - Adaptive practice and homework utilizing recommended resources
    - Encourage students to make transformations - use a common task or item in a different way
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge.
    - Extended time for revisions or opportunity to identify and develop areas of personal interest
- 504 Students
    - Video tutorials for each lesson with units for students to access as needed.
    - Mathematical literacy and vocabulary activities that provide scaffolded support for students to build vocabulary
    - Encourage creative expression and thinking by allowing students to choose how to approach a problem or assignment.

- Gifted & Talented Students
  - Enrichment activities for teachers to utilize when learners are advanced within the given standard/topic.
  - Encourage students to explore concepts in depth and encourage independent studies or investigations.
  - Modeling or independent student led research

Resources	
Savaas envision textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & EngageNY https://www.engageny.org/	
Suggested Time Frame:	College Prep: 44-46 days, Advanced: 34-36 days, Honors: 28-30 days

Content Area:	High School Mathematics - Geometry	Grade(s) 9-12
Unit Plan Title:	UNIT 4 - Polygons and Circles	

Domain(s)

G-CO Congruence
G-C Circles
G-SRT Similarity, Right Triangles, and Trigonometry
G-GPE Expressing Geometric Properties with Equation

Overview/Rationale

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension. A pair of perpendicular axes associates pairs of number with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa.

Content Standard(s)

All levels:
G-CO.C.9 Prove theorems about lines and angles
G-CO.C.10 Prove theorems about triangles
G-CO.C.11 Prove theorems about parallelograms
G-C.A.3 Construct the inscribed and circumscribed circles of a triangles, and prove properties of angles for a quadrilateral inscribed in a circle
G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures
G-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles

Advanced/Honors:
G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc
G-GPE.A.1 Derive the equation of a circle of given center and radius using Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation
G-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords
G-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector
Honors Only:
G-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically

G-GPE.A.2 Derive the equation of a parabola given a focus and directrix

Math Practice Standards

MP1 Make sense of problems and persevere in solving them.
MP3 Construct viable arguments and critique the reasoning of others.
MP4 Model with mathematics.
MP5 Use appropriate tools strategically.
MP6 Attend to precision.
MP7 Look for and make use of structure.
MP8 Look for and express regularity in repeated reasoning.

Integration of Technology

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

Interdisciplinary Standard(s)

RST.9-10.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.5 Analyze the relationships among concepts in a text, including relationships among key terms.

Enduring Understandings:

- Properties of geometric figures can be verified.
- The measures of geometric figures can be calculated and analyzed using a variety of strategies, tools, and technologies.
- Geometric figures can be represented in the coordinate plane.
- Relationships exist among the radius, secant, tangent, and chords of a circle.

Essential Question(s):

All levels

- What is the relationship between a segment and the points on its perpendicular bisector? Between an angle and the points on its bisector?
- What are the properties of the perpendicular bisectors in a triangle? What are the properties of the angle bisectors in a triangle?
- What are the properties of the medians in a triangle? What are the properties of the altitudes in a triangle?
- What are some relationships between the sides and angles of any triangle?
- When two triangles have two pairs of congruent sides, how are the third pair of sides and the pair of angles opposite the third pair of sides related?
- How does the number of sides in a convex polygon relate to the sums of the measures of the exterior and interior angles?
- How are diagonals and angle measures related in kites and trapezoids?
- What are the relationships of the sides, the angles, and the diagonals of a parallelogram?
- Which properties determine whether a quadrilateral is a parallelogram?
- What properties of rhombuses, rectangles, and squares differentiate them from other parallelograms?
- Which properties of the diagonals of a parallelogram help you classify a parallelogram?
- How are properties of geometric figures represented in the coordinate plane?

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Advanced/Honors

- How is the equation of a circle determined in the coordinate plane?
- How are arc length and sector area related to circumference and area of a circle?
- How is a tangent line related to the radius of a circle at the point of tangency?
- How are chords related to their central angles and intercepted arcs?
- How is the measure of an inscribed angle related to its intercepted arc?
- How are the measures of angles, arcs, and segments formed by intersecting secant lines related?

Honors Only

- How does the geometric description of a parabola relate to its equation?

Career Education:
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas. **9.4.12.CT.1:** Identify problem-solving strategies used in the development of an innovative product or practice. **9.4.12.IML.3:** Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions.

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)	
Students will know…. Key Vocabulary <u>All levels</u> ● Altitude ● Centroid ● Circumcenter ● Circumscribed ● Concurrent ● Equidistant ● Incenter ● Inscribed ● Median ● Orthocenter ● Point of concurrency ● Midsegment of a trapezoid <u>Advanced/Honors</u> ● Arc length Central angle Chord Inscribed angle Intercepted arc	***Students will be able to (do):*** All levels ● prove geometric theorems ● prove theorems involving similarity ● understand and apply theorems about circles ● experiment with transformations in the plane Advanced/Honors ● find arc lengths and areas of sectors of circles Honors only ● use coordinates to prove simple geometric theorems algebraically ● translate between the geometric descriptions and the equation for a conic section

Major arc Minor arc Point of tangency Radian Secant Sector of a circle Segment of a circle Tangent to a circle Honors Only Directrix Focus parabola	

Assessment Evidence:	
Alternative Assessments: Teachers will expose students to higher-order thinking problems that are similar to the NJSLA/NJGPA. These tasks will be used as additional support when covering the material in the unit. Students should receive feedback from the teacher on their performance on Performance Tasks. **Domain: G-CO** Congruence G-CO.A.1 G-CO.C.9 G-CO.C.10 **Domain: G-C** Circles G-C.A.2 G-C.A.3	**Assessments (Diagnostic, Formative, Summative, Benchmark, Alternative)** ● Diagnostic: Course Readiness Assessments, Topic Readiness Assessments ● Formative: Independent problems during lessons, Lesson Quizzes ● Summative: Topic Assessments, Performance Assessments, Benchmark and Mid-Course Assessments, End of Course Assessments ● Alternative- one on one student conferencing and verbal understanding of concepts

G-C.B.5 **Domain: G-SRT** Similarity, Right Triangles, and Trigonometry G-SRT.B.5 **Domain: G-GPE** Expressing Geometric Properties with Equation G-GPE.A.1 G-GPE.A.2 G-GPE.B.4 G-GPE.B.7	

Teaching and Learning Actions	
	Note: See attached list of lessons that should be taught throughout this unit. If using another resource, ***please align content to what is identified in this plan.***
Instructional Strategies and Activities Review Games (e.g. Jeopardy)	By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment.
Guided Notes	Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems.
Higher-order Thinking Problems	As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting.

Activators	Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson.
Closure	Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson

Accommodations

- Special Education Students
    - Ongoing Interventions, strategic Interventions and Intensive Interventions that are teacher and/or technologically driven within the curriculum and targeted to standards in need of support.
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge.
    - Extended time for revisions or opportunity to identify and develop areas of personal interest
- English Language Learners
    - Bilingual language glossary for mathematical literacy and vocabulary prior to and during each lesson.
    - English Language Learner support embedded into each lesson for use during the instructional part of the lesson (rephrasing, models, photos,etc.)
    - Targeted support strategies that focus on key areas of language development: listening, speaking, reading, and writing for students at different proficiency levels.
    - Technology supported assignments aligned with standards in students native language.
- Students at Risk of School Failure
    - Teacher led reteach to build understanding which encompass guided reteaching activities for teachers to utilize when students need a better understanding of lesson topics.
    - Readiness assessment individualized study plan generated based on a students' score on the topic readiness assessment. Allows students to gain additional support in standards that should have been mastered in a previous grade.
    - Adaptive practice and homework utilizing recommended resources
    - Encourage students to make transformations - use a common task or item in a different way
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge.
    - Extended time for revisions or opportunity to identify and develop areas of personal interest
- 504 Students
    - Video tutorials for each lesson with units for students to access as needed.
    - Mathematical literacy and vocabulary activities that provide scaffolded support for students to build vocabulary
    - Encourage creative expression and thinking by allowing students to choose how to approach a problem or assignment.
- Gifted & Talented Students
    - Enrichment activities for teachers to utilize when learners are advanced within the given standard/topic.
    - Encourage students to explore concepts in depth and encourage independent studies or investigations.

○ Modeling or independent student led research

Resources	
Savaas envision textbook and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/ , & EngageNY https://www.engageny.org/	
Suggested Time Frame:	College Prep: 34-36 days, Advanced: 25-27 days, Honors: 28-30 days

21ST CENTURY MATH APPLICATIONS CURRICULUM

Middle Township Public Schools
216 S. Main Street
Cape May Court House, NJ 08210

Introduction

This document serves to meet all requirements for curriculum as per the Middle Township Board of Education and the New Jersey Department of Education and will serve as a guide for lesson planning. Units within the curricular framework for mathematics are designed to be taught in the order in which they are presented. There is a logical and developmentally-appropriate progression of standards, with strong consideration given to Major, Supporting, and Additional content standards presented since most concepts build upon each other. Within the units, the teachers have flexibility of what order to present the standards. Major, Supporting and Additional clusters of mathematics content standards are based on the New Jersey Student Learning Standards. Suggested Mathematical Practice Standards are listed in each unit to be imbedded regularly in daily math instruction.

Course Description

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more adv anced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equi valent way of writing the expression that exhibits some different aspect of its meaning. For example, *p +* 0.05p can be interpreted as the addition of a 5% tax to a price *p.* Rewriting *p +* 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the convention s of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, *p +* 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equ ation.

An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables f orm a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strat egic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x+ 1= 0$ is an integer, not a whole number; the solution of $2x + 1= 0$ is a rational number, not an integer; the solutions of $x^2 — 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.
The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b2)/2)h,$ can be solved for h using the same de Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of t hese is an essential skill in modeling.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flex ibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). The eight standards for mathematical practices are as follows:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathemat ics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin wit h the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less lik ely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In sho rt, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards, which set an expectation of understanding, are potential "points of intersection" b etween the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematic

Pacing Guide

UNIT TITLE	ENDURING UNDERSTANDINGS	NJSLS	TIMEFRAME
Problem Solving in All Units	Understanding of what has occurred, why the process was effective, and can justify this with sound reasoning and evidence. Multiple strategies may be used to solve problems, although some are more effective than others. Reasonableness of solutions is determined by the context of the problems.	***Standards for Mathematical Practice MP1 through MP8***	Integrated Throughout Units/Course
1 - Income/Banking/Expenses	The way data is collected, organized, and displayed influences interpretation. Real world situations can be represented algebraically and graphically.	**N.RN.A.1 & 2** **N.RN.B.3** **A.SSE.A.1, 1a, & 1b** **A.SSE.B.3 & 3c** **A.CED.A.1 & 4** **A.REI.B.3** **F.IF.A.2** **F.IF.B.4** **F.IF.C.7 & 7a** **F.IF.C.8 & 8b** **F.BF.A.1 & 1a**	11 weeks (MP 1-2)
2 – Homeownership/Budget	The way data is collected, organized, and displayed influences interpretation. Real world situations can be represented algebraically and graphically.	**N.RN.A.1** **N.Q.A.2** **N.VM.C.6** **A.SSE.A.1 & 1b** **A.CED.A.2 & 3** **A.REI.D.10** **F.IF.B.4 & 5** **F.IF.C.7 & 7a**	9-10 weeks (MP 2-3)

		F.IF.C.8b **F.BF.A.1** **F.LE.A.1** **A.APR.D.6** **G.C.B.5** **S.ID.B.6, 6a,6b, & 6c** **S.ID.C.8**	
3 - Credit/Purchases	The way data is collected, organized, and displayed influences interpretation. Real world situations can be represented algebraically and graphically.	**N.RN.A.1** **N.Q.A.1 & 2** **A.SSE.A.1, 1b, & 2** **A.SSE.B.3 & 3c** **A.CED.A.2, 3, & 4** **F.IF.A.1 & 2** **F.IF.B.6** **F.IF.C.7, 7a, 7b, 7e, 8b, & 9** **F.BF.A.1 & 1a** **F.LE.A.1, 1b, & 1c** **F.LE.B.5** **G.C.B.5** **S.ID.A.2 & 3** **S.ID.B.6, 6a, & 6b** **S.ID.C.7**	9 weeks (MP 3-4)
4 - **I**nvestments/Retirement	The way data is collected, organized, and displayed influences interpretation. Real world situations can be represented algebraically and graphically.	**N.Q.A.1, 2, &3** **A.APR.B.3** **A.REI.A.2** **A.REI.B.3, 4, & 4b** **A.REI.C.6 & 7** **A.REI.D.10, 11, & 12**	6 weeks (MP 4)

		A.SSE.A.1, 1a, & 1b **A.SSE.B.3** **A.CED.A.1, 2, 3, & 4** **F.IF.A.1** **F.IF.B.4 & 5** **F.IF.C.7, 7a, 8, & 8b** **S.MD.A.1, 2, & 4** **S.MD.B.5**	

Unit Plans

Content Area:	21st Century Math Applications	Grade(s): 9-12
Unit Plan Title:	Unit 1: Income/Banking/Expenses	

Domain(s)

N-RN; A-SSE; A-CED; F-IF; F-BF; A-REI; F-LE

Overview/Rationale

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

Equations & Inequalities plus Creating Equations: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Building Functions, Interpreting Functions and Linear & Exponential Models: Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Interpreting Functions : Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

The Real Number Systems: In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by imaginary numbers to form complex numbers.

Quantities: Quantities has a real application to real world problems. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Content Standard(s) (Established Goals)

N-RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.*
R-RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.
R-RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context
A-SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients.
A-SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P*(1+*r*)n *as the product of P and a factor not depending on P.*
A-SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
A-SSE.B.3c: Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15t *can be rewritten as* (1.151/12)12*t* ≈ 1.01212t *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
A.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.
A.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
A.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
F.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F.IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.
F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF.C.8b: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
F.BF.A.1: Write a function that describes a relationship between two quantities.★
F.BF.A.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.

8.1.5.A.3: Use graphic organizers to organize information about problem or issue.

8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.

8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.

8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.

8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.

8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.

8.2.2.E.1: List and demonstrate the steps to an everyday task**.**

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

Students will understand that…

The way data is collected, organized, and displayed influences interpretation.

Real world situations can be represented algebraically and graphically.

Essential Question(s) : (What provocative questions will foster inquiry, understanding, and transfer of learning?)

How do I use algebraic equations to analyze or solve a problem?

What is the difference between an Arithmetic sequence and Geometric sequence?

What is compound interest? Give a real life situation that would involve compound interest.

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply.		**21st Century Themes**	*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*		**21st Century Skills**
		Global Awareness		**E,T,A**	**Critical Thinking & Problem Solving**
		Environmental Literacy		**E,T**	**Creativity and Innovation**
		Health Literacy		**E,T**	**Collaboration, Teamwork and Leadership**
		Civic Literacy		**E,T**	**Cross-Cultural and Interpersonal Communication**
		Financial, Economic, Business and Entrepreneurial Literacy		**E,T**	**Communication and Media Fluency**
				E,T	**Accountability, Productivity and Ethics**

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

	E	**CRP1. Act as a responsible and contributing citizen and employee**
	E,T,A	**CRP2. Apply appropriate academic and technical skills**
	E	**CRP3. Attend to personal health and financial well-being**
	E,T,A	**CRP4. Communicate clearly and effectively with reason**
	E	**CRP5. Consider the environmental, social and economic impacts of decisions**

E,T	**CRP6. Demonstrate creativity and innovation**
E	**CRP7. Employ valid and reliable research strategies**
E,T,A	**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them**
E,T	**CRP9. Model integrity, ethical leadership and effective management**
E	**CRP10. Plan education and career paths aligned to personal goals**
E,T	**CRP11. Use technology to enhance productivity**
E	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
- The difference between essential and discretionary expenses - The difference between mean, median, and mode - The difference between cumulative and relative frequency - The difference between measures of central tendency and measures of dispersion - How to interpret patterns and trends based on date - The difference between simple and compound interest - The difference between a registry and a statement - How to compare benefits of various bank offers - The difference between Arithmetic sequence and Geometric sequence - Advantages and disadvantages of pay based on production - How to interpret and use the information on a pay stub, W-2 form, and 1099 form - The difference between a tax credit and a tax deduction	- Determine the mean, median, and mode of a data set - Create and interpret a frequency distribution table - Determine and interpret cumulative and relative frequency - Compute the range, mean deviation, and variance for a set of data - Construct a scatter plot. - Graph bivariate data - Sequence the difference between a registry and a statement - Sequence an ongoing balance of a check registry - Contrast the difference between simple and compound interest - Compute periodic salary based on annual contract salary

	- Express classified as prices as piecewise functions - Compute weekly, semimonthly, and biweekly earnings given annual salary - Compute hourly pay and overtime pay given hourly rate - Compute annual salaries based on annual percentage increases - Compute pay based on percent commission and piecework pay - Compute paycheck deductions for Social Security ans Medicare - Compute federal income taxes using a tax table and tax schedules - Complete Form 1040EZ and 1040A
Assessment Evidence:	
Performance Tasks: *(Through what authentic performance tasks will students demonstrate the desired understandings? By what criteria will performances of understanding be judged?)* **Students will complete teacher made assignments and projects throughout the unit. Activities and projects are suggested in the Financial Algebra teacher resource kit.** **Other Resource Activities:** **https://www.moneyinstructor.com/lessonplan.asp** **https://www.education.ne.gov/ndesearch.html?keywords=the+game+of+life-on+a+budget&submit=Search**	**Other Assessment Measures:** *(Through what other evidence (E.g. quizzes, tests, academic prompts, observations, homework, journals, etc.) will students demonstrate achievement of the desired results? How will students reflect upon and self- assess their learning?)* ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.*** • Tests • Quizzes • Homework Assignments • Classwork Assignments

https://learning.blogs.nytimes.com/2011/04/11/no-taxation-without-calculation-filling-out-tax-returns/ https://www.mathworksheetsland.com/.../business/21statetax1.pdf	
Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?	
Instructional Strategies and Activities *D*	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Financial Algebra by CENGAGE Learning. If using another resource, please align content to what is identified in this plan.*** By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment. Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems. As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting. Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson. Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson.
Resources	
Financial Algebra by CENGAGE Learning and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/, & Engage NY https://www.engageny.org/	
Suggested Time Frame:	11 weeks (MP 1-2)

Content Area:	21st Century Math Applications	Grade(s): 9-12
Unit Plan Title:	Unit 2: Homeownership/Budget	

Domain(s)

N-RN; N-Q; N-VM; A-SSE; A-CED; A-APR; F-IF; F-BF; A-REI; F-LE; G.C; S-ID;

Overview/Rationale

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

Equations & Inequalities plus Creating Equations: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Building Functions, Interpreting Functions and Linear & Exponential Models: Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

Interpreting Functions : Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

The Real Number Systems: In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by imaginary numbers to form complex numbers.

Quantities: Quantities has a real application to real world problems. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Content Standard(s) (Established Goals)

N-RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.*

N-Q.A.2: Define appropriate quantities for the purpose of descriptive modeling.

N-VM.C.6: (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context

A-SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P*(1+*r*)n *as the product of P and a factor not depending on P.*

A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A.REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.IF.C.8b: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
F.BF.A.1: Write a function that describes a relationship between two quantities.★
F.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.
A-APR.D.6: Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
G-C.B.5: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
S-ID.B.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
S-ID.B.6a: Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.*
S-ID.B.6b: Informally assess the fit of a function by plotting and analyzing residuals.
S-ID.B.6c: Fit a linear function for a scatter plot that suggests a linear association.
S-ID.C.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.

8.1.5.A.3: Use graphic organizers to organize information about problem or issue.

8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.

8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.

8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.

8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.

8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.

8.2.2.E.1: List and demonstrate the steps to an everyday task**.**

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

Students will understand that…

The way data is collected, organized, and displayed influences interpretation.

Real world situations can be represented algebraically and graphically.

Essential Question(s) : (What provocative questions will foster inquiry, understanding, and transfer of learning?)

How is geometry used when making a floor plan?

What do you need to know about mortgages?

How much will it cost to run utilities in your home?

What do you need to consider when creating a budget?

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply. **21st Centur y Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Centur y Skills**	
	Global Awareness	**E,T,A**	**Critical Thinking & Problem Solving**
	Environmental Literacy	**E,T**	**Creativity and Innovation**
	Health Literacy	**E,T**	**Collaboration, Teamwork and Leadership**
	Civic Literacy	**E,T**	**Cross-Cultural and Interpersonal Communication**
	Financial, Economic, Business and Entrepreneurial Literacy	**E,T**	**Communication and Media Fluency**
		E,T	**Accountability, Productivity and Ethics**

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

E	**CRP1. Act as a responsible and contributing citizen and employee**
E,T,A	**CRP2. Apply appropriate academic and technical skills**
E	**CRP3. Attend to personal health and financial well-being**
E,T,A	**CRP4. Communicate clearly and effectively with reason**
E	**CRP5. Consider the environmental, social and economic impacts of decisions**
E,T	**CRP6. Demonstrate creativity and innovation**
E	**CRP7. Employ valid and reliable research strategies**
E,T,A	**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them**
E,T	**CRP9. Model integrity, ethical leadership and effective management**
E	**CRP10. Plan education and career paths aligned to personal goals**
E,T	**CRP11. Use technology to enhance productivity**
E	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
- How to determine the relationship between square footage and monthly rent - What formulas will find the area and volume of regular and irregular polygons - How to explain the necessary research needed before purchasing a home - Explain the advantage and disadvantages of purchasing different types of homes - Explain the pros and cons of buying a home verses renting a place to live	- Calculate the affordability of a monthly rent - Determine lease signing costs - Calculate moving expenses - Compute the perimeter and the area of a polygon - Compute areas of irregular regions - Compute volumes of rectangular solids - Compute the monthly cost of paying for a house - Estimate costing costs

- How to use trigonometry when completing home maintenance and improvements - What it takes financial to run a household - How to set up, graph, and interpret an average cost function - Compare different plans for utilities - To visualize and interpret a budget using a circle graph, a bar, graph, a line, graph, and a budget line graph - How to interpret a cash flow chart, frequency budget plan, and year-long expense budget plan	- Create an amortization table for a fixed-rate mortgage plus extra payments - Investigate the amortization table for an adjustable-rate mortgage - Calculate discount points for a mortgage - Determine the breakeven time for discount points - Calculate negative points - Compute the costs of purchasing a cooperative or a condominium - Find the missing sides of a right triangle using the Pythagorean theorem - Use the converse of the Pythagorean theorem - Find the slope of a line - Describe proportions in similar triangles - Find the missing side of a right triangle using trigonometry - Find the missing angles in right triangles using inverse trigonometric functions - Compute the cost of electricity, gas, oil, and water for a home - Develop a cash flow chart, frequency budget plan, and year-long expense budget plan - Add, subtract, and multiply matrices - Multiply a matrix by a scalar
Assessment Evidence:	

Performance Tasks: *(Through what authentic performance tasks will students demonstrate the desired understandings? By what criteria will performances of understanding be judged?)* **Students will complete teacher made assignments and projects throughout the unit. Activities and projects are suggested in the Financial Algebra teacher resource kit.** **Other Resource Activities:** **https://www.moneyinstructor.com/lessonplan.asp**	**Other Assessment Measures:** *(Through what other evidence (E.g. quizzes, tests, academic prompts, observations, homework, journals, etc.) will students demonstrate achievement of the desired results? How will students reflect upon and self- assess their learning?)* ******Attach all Benchmarks*** ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.*** • Tests • Quizzes • Homework Assignments • Classwork Assignments
Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?	
Instructional Strategies and Activities *D*	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Financial Algebra by CENGAGE Learning. If using another resource, please align content to what is identified in this plan.*** By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment. Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems. As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting. Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson. Each day teachers should close the lesson with a question or problem. This will help students review all that was

	taught in the lesson.
Resources	
Financial Algebra by CENGAGE Learning and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/, & Engage NY https://www.engageny.org/	
Suggested Time Frame:	9-10 weeks (MP 2-3)

Content Area:	21st Century Math Applications	Grade(s): 9-12
Unit Plan Title:	Unit 3: Credit/Purchases	

Domain(s)

N-Q; RN-A; A-SSE; A-CED; F-IF; F-BF; F.LE; G.C; S-ID;

Overview/Rationale

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

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Interpreting Functions : Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

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quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Content Standard(s) (Established Goals)

N-RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.*

N-Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2: Define appropriate quantities for the purpose of descriptive modeling.

A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context

A-SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P*(1+*r*)n *as the product of P and a factor not depending on P.*

A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it. *For example, see x*4 – *y*4 *as* (*x*2)2 – (*y*2)2, *thus recognizing it as a difference of squares that can be factored as* (*x*2 – *y*2)(*x*2 + *y*2).

A-SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-SSE.B.3c: Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15t *can be rewritten as* (1.151/12)12*t* ≈ 1.01212t *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

F.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

F.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
F.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F.IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.
F.IF.C.7b: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
F.IF.C.7e: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F.IF.C.8b: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
F.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
F.BF.A.1: Write a function that describes a relationship between two quantities.★
F.BF.A.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.
F.LE.A.1b: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
F.LE.A.1c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
F.LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.
G-C.B.5: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
S-ID.A.2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S-ID.A.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
S-ID.B.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
S-ID.B.6a: Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.*
S-ID.B.6b: Informally assess the fit of a function by plotting and analyzing residuals.
S-ID.C.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.

8.1.5.A.3: Use graphic organizers to organize information about problem or issue.

8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.

8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.

8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.

8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.

8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.

8.2.2.E.1: List and demonstrate the steps to an everyday task**.**
Interdisciplinary Standard(s)
NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content. **NJSLSA.W4:** Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)
Students will understand that… The way data is collected, organized, and displayed influences interpretation. Real world situations can be represented algebraically and graphically.
Essential Question(s) : (What provocative questions will foster inquiry, understanding, and transfer of learning?)
What do you need to know before using credit? What information do you need to know before taking out a loan? What do I need to know to use credit cards? How can statistics help you negotiate the sales or purchase of a car? How does probability affect your auto insurance? How does your car lose its value?
In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply. 21st Centur y Themes			*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* 21st Centur y Skills	
	Global Awareness		E,T,A	Critical Thinking & Problem Solving
	Environmental Literacy		E,T	Creativity and Innovation
	Health Literacy		E,T	Collaboration, Teamwork and Leadership
	Civic Literacy		E,T	Cross-Cultural and Interpersonal Communication
	Financial, Economic, Business and Entrepreneurial Literacy		E,T	Communication and Media Fluency
			E,T	Accountability, Productivity and Ethics

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

	E	CRP1. Act as a responsible and contributing citizen and employee
	E,T,A	CRP2. Apply appropriate academic and technical skills
	E	CRP3. Attend to personal health and financial well-being
	E,T,A	CRP4. Communicate clearly and effectively with reason
	E	CRP5. Consider the environmental, social and economic impacts of decisions
	E,T	CRP6. Demonstrate creativity and innovation
	E	CRP7. Employ valid and reliable research strategies
	E,T,A	CRP8. Utilize critical thinking to make sense of problems and persevere in solving them
	E,T	CRP9. Model integrity, ethical leadership and effective management

E	**CRP10. Plan education and career paths aligned to personal goals**
E,T	**CRP11. Use technology to enhance productivity**
E	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
- Basic credit terms and identify types of lending institutions - How to read monthly payments from a table - The options available for student loans - Logarithmic loan length formula - The basic vocabulary of credit cards - Average daily balance - How to read a frequency distribution from a set of data, a box-and –whisker plot, and stem-and leaf plot - The different types of auto insurance coverage - How to read Venn diagrams - The distance formula - The yaw mark formula	- Compute finance charges for installment purchases - Compute monthly payments using a formula - Compute interest on loans - Calculate the interest due in various student loan situations - Apply the simplifies daily interest formula - Determine the interest on a loan given the principal, time, and APR - Apply the Logarithmic Loan length formula to determine the term of a loan - Use the regression to find the curve of best fit using data from a loan table - Compute an average daily balance - Calculate the average daily balance and finance charges using the credit calendar - Compute the cost of classified ads for used cars - Compute the cost of sales tax on automobiles - Create a frequency distribution, box-and-whisker plot, and stem-and –leaf plot when given a set of data - Compute insurance costs and payments on insurance

	claims - Create Venn diagrams - Write, interpret and graph a straight line depreciation equation and exponential deprecation equation - Write, interpret, and use the distance formula - Calculate reaction time and distance in the English Standard system - Apply the yaw mark formula
Assessment Evidence:	
Performance Tasks: *(Through what authentic performance tasks will students demonstrate the desired understandings? By what criteria will performances of understanding be judged?)* **Students will complete teacher made assignments and projects throughout the unit. Activities and projects are suggested in the Financial Algebra teacher resource kit.** **Other Resource Activities:** **https://www.moneyinstructor.com/lessonplan.asp** **https://www.lessonplanet.com/lesson-plans/buying-a-car/all** **https://alex.state.al.us/lesson_view.php?id=24803**	**Other Assessment Measures:** *(Through what other evidence (E.g. quizzes, tests, academic prompts, observations, homework, journals, etc.) will students demonstrate achievement of the desired results? How will students reflect upon and self- assess their learning?)* ******Attach all Benchmarks*** ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.*** • Tests • Quizzes • Homework Assignments • Classwork Assignments
Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?	
Instructional Strategies and Activities	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Financial Algebra by CENGAGE Learning. If using another resource, please align content to what is identified in this plan.***

D	By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment. Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems. As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting. Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson. Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson.
Resources	
Financial Algebra by CENGAGE Learning and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/, & Engage NY https://www.engageny.org/	
Suggested Time Frame:	9 weeks (MP 3-4)

Content Area:	21st Century Math Applications	Grade(s): 9-12
Unit Plan Title:	Unit 4: Investments/Retirement	

Domain(s)

N-Q; A-SSE; A-REI; A-CED; A-APR F-IF; F-BF; S-ID; S.MD

Overview/Rationale

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning.

Equations & Inequalities plus Creating Equations: An equation is a statement of equality between two expressions. An equation can often be solved by successively deducing from it one or more simpler equations. Strategic competence in solving included looking ahead for productive manipulations and anticipating the nature and number solutions. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

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Interpreting Functions : Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

The Real Number Systems: In high school, students will be exposed to yet another extension of numbers, when the real numbers are augmented by imaginary numbers to form complex numbers.

Quantities: Quantities has a real application to real world problems. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In high school, students encounter a wider variety of units in modeling,

e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

Content Standard(s) (Established Goals)

N-Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N-Q.A.2: Define appropriate quantities for the purpose of descriptive modeling.
N-Q.A.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities

A-APR.B.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
A-REI.A.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A-REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A-REI.B.4: Solve quadratic equations in one variable.
A-REI.B.4b: Solve quadratic equations by inspection (e.g., for $x2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
A-REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
A-REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = –3x$ and the circle $x2 + y2 = 3$.
A-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A-REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
A-REI.D.12: Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.
A-SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients.
A-SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)n$ *as the*

product of P and a factor not depending on P.
A-SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
A.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.
A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
A.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
F.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★
F.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
F.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F.IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.
F.IF.C.8:
F.IF.C.8b: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*
F.BF.A.1: Write a function that describes a relationship between two quantities.★
F.BF.A.1b: Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
S-ID.C.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-MD.A.1: (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
S-MD.A.2: (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution
S-MD.A.4: (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
S-MD.B.5: (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

Math Practice Standards

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

Standards for Technology

8.1.5.A.1: Select and use the appropriate digital tools and resources to accomplish a variety of tasks including solving problems.

8.1.5.A.3: Use graphic organizers to organize information about problem or issue.

8.2.2.C.1: Brainstorm ideas on how to solve a problem or build a product.

8.2.5.C.4: Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.

8.2.8.C.4: Identify the steps in the design process that would be used to solve a designated consequences of other technologies and present your findings.

8.2.2.D.1: Identify and collect information about a problem that can be solved by technology, generate ideas to solve the problem, and identify constraints and trade –offs to be considered.

8.2.5.D.3: Follow step by step directions to assemble a product or solve a problem.

8.2.2.E.1: List and demonstrate the steps to an everyday task**.**

Interdisciplinary Standard(s)

NJSLSA.W2: Write information/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

NJSLSA.W4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

Enduring Understandings: (What are the big ideas? What specific understandings about them are desired? What misunderstandings are predictable?)

Students will understand that…

The way data is collected, organized, and displayed influences interpretation.

Real world situations can be represented algebraically and graphically.

Essential Question(s) : (What provocative questions will foster inquiry, understanding, and transfer of learning?)

What is the Stock Market and how does it work?

What do I need to know when planning for retirement?

In this unit plan, the following 21st Century themes and skills are addressed:

Check all that apply.		**21st Century Themes**	*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*	**21st Century Skills**
		Global Awareness	**E,T,A**	**Critical Thinking & Problem Solving**
		Environmental Literacy	**E,T**	**Creativity and Innovation**
		Health Literacy	**E,T**	**Collaboration, Teamwork and Leadership**
		Civic Literacy	**E,T**	**Cross-Cultural and Interpersonal Communication**
		Financial, Economic, Business and Entrepreneurial Literacy	**E,T**	**Communication and Media Fluency**
			E,T	**Accountability, Productivity and Ethics**

In this unit plan, the following Career Ready Practices are addressed:

*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*

	E	**CRP1. Act as a responsible and contributing citizen and employee**
	E,T,A	**CRP2. Apply appropriate academic and technical skills**
	E	**CRP3. Attend to personal health and financial well-being**
	E,T,A	**CRP4. Communicate clearly and effectively with reason**
	E	**CRP5. Consider the environmental, social and economic impacts of decisions**
	E,T	**CRP6. Demonstrate creativity and innovation**

E	**CRP7. Employ valid and reliable research strategies**
E,T,A	**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them**
E,T	**CRP9. Model integrity, ethical leadership and effective management**
E	**CRP10. Plan education and career paths aligned to personal goals**
E,T	**CRP11. Use technology to enhance productivity**
E	**CRP12. Work productively in teams while using cultural global competence**

Student Learning Goals/Objectives: (What key knowledge and skills will students acquire as a result of this unit? What should they eventually be able to do as a result of such knowledge and skill?)

Students will know….	***Students will be able to (do)…***
- Basic vocabulary of business organizations - How to use stock data to follow the daily progress of corporate stock - How to interpret stock bar graphs (stock bar chart & Stock candlestick chart) - To explain how data is smoothed - How to explain stock market ticker information - How to determine the total value of a trade and trade volumes from ticker information - Basic vocabulary of buying and selling shares - Have an understanding of the concept of shareowners splitting the profit of the corporation they own - How to choose sample subjects without bias and how to reduce bias in experiments - How to determine if a statistic is biased or unbiased - Supply and demand curves - Fixed and variable expenses	- Compute financial responsibility of business ownership based on ratios and percentages - Create a stock bar chart & stock candlestick chart - Calculate simple moving averages using the arithmetic average formula, plus using subtraction and addition - Compute gains and losses from stock trades - Calculate the post –split outstanding shares and share price for a traditional and reverse split - Calculate the fractional value amount that a shareholder receives after a split - Compute dividend income, yields for a given stock, and interest earned on corporate bonds - Create diagrams for experimental designs - Compute combinations and unbiased estimators - Create an expense equation based on fixed and variable expenses

- Graphs of expense and revenue function - Profit equation - Retirement income from savings - Social Security Benefits - Pensions - Life insurance	- Write, graph, and interpret the expense and revenue function - Determine a profit equation given the expense and revenue functions - Calculate future values of retirement investments - Compute federal income tax on benefits that are paid under Social Security calculate pension benefits during and after vesting periods - Compute the advantages and disadvantages of different types of life insurance
Assessment Evidence:	
Performance Tasks: *(Through what authentic performance tasks will students demonstrate the desired understandings? By what criteria will performances of understanding be judged?)* **Students will complete teacher made assignments and projects throughout the unit. Activities and projects are suggested in the Financial Algebra teacher resource kit.** **Other Resource Activities:** **https://www.moneyinstructor.com/investing.asp**	**Other Assessment Measures:** *(Through what other evidence (E.g. quizzes, tests, academic prompts, observations, homework, journals, etc.) will students demonstrate achievement of the desired results? How will students reflect upon and self- assess their learning?)* ******Attach all Benchmarks*** ******All benchmarks will collect at the end of each marking period. A blank copy and graded benchmarks will be turned into the department supervisor.*** • Tests • Quizzes • Homework Assignments • Classwork Assignments
Teaching and Learning Actions: (What learning experiences and instruction will enable students to achieve the desired results?	
Instructional Strategies and	***Note: See attached list of lessons that should be taught throughout this unit. The list follows the chapters and lessons found in the Financial Algebra by CENGAGE Learning. If using another resource, please align content***

Activities *D*	***to what is identified in this plan.*** By using review games, the students will be able to practice concepts with their peers. They will also be able to review all of the major concepts from the unit before taking a formal assessment. Teachers should use guided or skeleton notes to help the students organize the content. These notes should include vocabulary, examples, and independent practice problems. As discussed in the Performance Task section, teachers will select at least one task from each domain that is covered in this unit. These problems/activities will help the students use the domain in an assessment setting. Each day teachers should open the lesson with an activator. This will help students review the content from the previous lesson. Each day teachers should close the lesson with a question or problem. This will help students review all that was taught in the lesson.
Resources	
Financial Algebra by CENGAGE Learning and resources, NJCTL, Illustrative Mathematics https://www.illustrativemathematics.org/, & Engage NY https://www.engageny.org/	
Suggested Time Frame:	6 weeks (MP 4)

Curricular Map of High School Student Learning Standards for Mathematics

	Algebra	Geometry	Algebra II	Pre calculus
NUMBER AND QUANTITY (N)				
The Real Number System (RN)				
N-RN.A.1.Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	X		X	X
N-RN.A.2.Rewrite expressions involving radicals and rational exponents using the properties of exponents.	X		X	X
N-RN.B.3.Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	X			X
Quantities (Q)				
N-Q.A.1.Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	X	X		X
N-Q.A.2.Define appropriate quantities for the purpose of descriptive modeling.	X	X	X	X
N-Q.A.3.Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	X			X
The Complex Number System (CN)				
N-CN.A.1.Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real.			X	
N-CN.A.2.Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.			X	
N-CN.A.3.(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.			X	X

	Algebra	Geometry	Algebra II	Pre calculus
N-CN.B.4.(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.				X
N-CN.B.5.(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.				X
N-CN.B.6.(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				X
N-CN.C.7.Solve quadratic equations with real coefficients that have complex solutions.			X	
N-CN.C.8.(+) Extend polynomial identities to the complex numbers.			X	X
N-CN.C.9.(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.			X	X
Vector and Matrix Quantities (VM)				
N-VM.A.1.(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v, 14 114,** v).		X		X
N-VM.A.2.(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				X
N-VM.A.3.(+) Solve problems involving velocity and other quantities that can be represented by vectors.				X

	Algebra	Geometry	Algebra II	Pre calculus
N-VM.B.4. (+) Add and subtract vectors. m. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. n. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. o. Understand vector subtraction v— was v + (—w), where —w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.		X		
N-VM.B.5. (+) Multiply a vector by a scalar. i. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vv) = (mix, cv_i). j. Compute the magnitude of a scalar multiple cv using licv11= iciv. Compute the direction of cv knowing that when Iclv # 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).				X
N-VM.C.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.				X
N-VM.C.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.				X
N-VM.C.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.				X
N-VM.C.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.				X
N-VM.C.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.				X

	Algebra	Geometry	Algebra II	Pre calculus
N-VM.C.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.				X
N-VM.C.12. (+) Work with 2 x 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.				X
Algebra (A)				
Seeing Structure in Expressions (SSE)				
A-SSE.A.1. Interpret expressions that represent a quantity in terms of its context.* i. Interpret parts of an expression, such as terms, factors, and coefficients. j. Interpret complicated expressions by viewing one or more of their parts as a single entity.	X	X	X	X
A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it.	X		X	X
A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* m. Factor a quadratic expression to reveal the zeros of the function it defines. n. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. o. Use the properties of exponents to transform expressions for exponential functions.	X	X	X	X
A-SSE.B.4. Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.			X	X
Arithmetic with Polynomials and Rational Expressions (APR)				
A-APR.A.1.Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	X		X	
A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	X		X	X

	Algebra	Geometry	Algebra II	Pre calculus
A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	X		X	X
A-APR.C.4. Prove polynomial identities and use them to describe numerical relationships.			X	
A-APR.C.5. (+) Know and apply the Binomial Theorem for the expansion of *(x* + y)" in powers of *x* and y for a positive integer *n,* where *x* and y are any numbers, with coefficients determined for example by Pascal's Triangle.			X	
A-APR.D.6.Rewrite simple rational expressions in different forms; write $a^{(}x^{)}/_{b(x)}$ in the form *q(x)* + $^{trx)}$/ *b(x),* where a(x), *b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x),* using inspection, long division, or, for the more complicated examples, a computer algebra system.	X		X	
A-APR.D.7.(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	X		X	
Creating Equations (CED)				
A-CED.A.1.Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	X			
A-CED.A.2.Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	X	X		
A-CED.A.3.Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	X		X	
A-CED.A.4.Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	X			X

	Algebra	Geometry	Algebra II	Pre calculus
Reasoning with Equations and Inequalities (REI)				
A-REI.A.1.Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**X**		**X**	**X**
A-REI.A.2.Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**X**		**X**	**X**
A-REI.B.3.Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**X**			**X**
A-REI.B.4. Solve quadratic equations in one variable. i. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. j. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b.*	**X**		**X**	**X**
A-REI.C.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	**X**			
A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	**X**		**X**	
A-REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.			**X**	
A-REI.C.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.				**X**
A-REI.C.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).				**X**

	Algebra	Geometry	Algebra II	Pre calculus
A-REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	X			
A-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y= *f(x)* and y = *g(x)* intersect are the solutions of the equation *f(x) = g(x);* find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	X	X		X
A-REI.D.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	X			
FUNCTIONS (F)				
Interpreting Functions (IF)				
F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x.* The graph of *f* is the graph of the equation y= *f(x).*	X			X
F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	X			X
F-IF.A.3.Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	X		X	
F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	X			X
F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it descrbes.*	X			X

	Algebra	Geometry	Algebra II	Pre calculus Math Topics
F-IF.B.6.Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	X		X	X
F-IF.C.7.Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* u. Graph linear and quadratic functions and show intercepts, maxima, and minima. v. Graph square root, cube root, and piecewise-defined functions, including step *functions* and absolute value functions. w. Graph polynomial functions, identifying zeros when suitable factorizations are *available,* and showing end behavior. *x.* (+) Graph rational functions, identifying zeros and asymptotes when suitable *factorizations* are available, and showing end behavior. y. Graph exponential and logarithmic functions, showing intercepts and end *behavior,* and trigonometric functions, showing period, midline, and amplitude.	X		X	X
F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. i. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. j. Use the properties of exponents to interpret expressions for exponential functions.	X		X	
F-IF.C.9.Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	X		X	
Building Functions (BF)				
F-BF.A.1.Write a function that describes a relationship between two quantities.* m. Determine an explicit expression, a recursive process, or steps for calculation from a context. n. Combine standard function types using arithmetic operations. o. (+) Compose functions.	X		X	X
F-BF.A.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	X		X	X

	Algebra	Geometry	Algebra II	Pre calculus
F-BF.B.3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	X		X	X
F-BF.B.4. Find inverse functions. q. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. r. (+) Verify by composition that one function is the inverse of another. s. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. t. (+) Produce an invertible function from a non-invertible function by restricting the domain.			X	X
F-BF.B.5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving exponents and logarithms.			X	X

	Algebra	Geometry	Algebra II	Pre calculus
Linear, Quadratic and Exponential Models (LE)				
F-LE.A.1.Distinguish between situations that can be modeled with linear functions and with exponential functions. m. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. n. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. o. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	X			X
F-LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	X		X	
F-LE.A.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	X			
F-LE.A.4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e;* evaluate the logarithm using technology.			X	X
F-LE.B.5.Interpret the parameters in a linear or exponential function in terms of a context.	X			
Trigonometric Functions (TF)				
F-TF.A.1.Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.			X	X
F-TF.A.2.Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.			X	X
F-TF.A.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for Tr13, Tr/4 and Tr/6, and use the unit circle to express the values of sine, cosines, and tangent for *x,* IF + *x,* and 2¯rr — *x* in terms of their values for *x,* where *x* is anyreal number.				X

	Algebra	Geometry	Algebra H	Pre calculus
F-TF.A.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.				X
F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*			X	X
F-TF.B.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.				X
F-TF.B.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*				X
F-TF.C.8. Prove the Pythagorean identity $\sin^2(0) + \cos^2(0) = 1$ and use it to find sin(0), cos(e), or tan(0) given sin(0), cos(0), or tan(0) and the quadrant of the angle.			X	X
F-TF.C.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.				X
GEOMETRY (G)				
Congruence (CO)				
G.CO.A.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.		X		
G-CO.A.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).		X		
G-CO.A.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.		X		
G-CO.A.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		X		
G-CO.A.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.		X		
G-CO.B.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.		X		
G-CO.B.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.		X		

	Algebra	Geometry	Algebra II	Pre calculus
G-CO.B.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.		X		
G-CO.C.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*		**X**		**X**
G-CO.C.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*		**X**		**X**
G-CO.C.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*		**X**		
G-CO.D.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*		**X**		
G-CO.D.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.		**X**		
Similarity, Right Triangles, and Trigonometry (SRT)				
G-SRT.A.1. Verify experimentally the properties of dilations given by a center and a scale factor: i. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. j. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.		**X**		
G-SRT.A.2.Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.		X		
G-SRT.A.3.Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	**X**	**X**		
G-SRT.B.4.Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*		**X**		

	Algebra	Geometry	Algebra II	Pre calculus
G-SRT.B.5.Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.		X		X
G-SRT.C.6.Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		X	X	X
G-SRT.C.7.Explain and use the relationship between the sine and cosine of complementary angles.		X		X
G-SRT.C.8.Use trigonometric ratios and Pythagorean Theorem to solve right triangles in applied problems.*	X	X		X
G-SRT.D.9.(+) Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.		X		X
G-SRT.D.10.(+) Prove the Laws of Sines and Cosines and use them to solve problems.		X		X
G-SRT.D.11.(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).		X		X
Circles (C)				
G-C.A.1.Prove that all circles are similar.		X		
G-C.A.2.Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*		*X*		
G-C.A.3.Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.		X		
G-C.A.4.(+) Construct a tangent line from a point outside a given circle to the circle.		X		
G-C.B.5.Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.		X		
Expressing Geometric Properties with Equations (GPE)				
G-GPE.A.1.Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		X		X
G-GPE.A.2.Derive the equation of a parabola given a focus and directrix.			X	X
G-GPE.A.3.(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	X			X

	Algebra	Geometry	Algebra II	Pre calculus
G-GPE.B.4.Use coordinates to prove simple geometric theorems algebraically.		X	X	
G-GPE.B.5.Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	X	X		
G-GPE.B.6.Find the point on a directed line segment between two given points that partitions the segment in a given ratio.		X		
G-GPE.B.7.Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*				
Geometric Measurement and Dimension (GMD)				
G-GMD.A.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*		X		X
G-GMD.A.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.		X		X
G-GMD.A.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*		X		X
G-GMD.B.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.		X		
Modeling with Geometry (MG)				
G-MG.A.1.Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*		X		X
G-MG.A.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*				
G-MG.A.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*		X		
Statistics and Probability (S)				
Interpreting Categorical and Quantitative Data (ID)				
S-ID.A.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	X			

	Algebra	Geometry	Algebra II	Pre calculus
S-ID.A.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	X			X
S-ID.A.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	X			X
S-ID.A.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	X			X
S-ID.B.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	X			X
S-ID.B.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. m. Fit a function to the data; use functions fitted to data (including with the use of technology) to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. n. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology o. Fit a linear function for a scatter plot that suggests a linear association.	X			X
S-ID.C.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	X			
S-I D.C.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	X			
S-ID.C.9. Distinguish between correlation and causation.	X			
Making Inferences and Justifying Conclusions (IC)				
S-IC.A.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.			X	X

	Algebra	Geometry	Algebra II	Pre calculus
S-IC.A.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.			X	X
S-IC.B.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.			X	
S-IC.B.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.			X	X
S-IC.B.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.			X	
S-IC.B.6. Evaluate reports based on data.			X	
Conditional Probability and the Rules of Probability (CP)				
S-CP.A.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").			X	X
S-CP.A,2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.			X	X
S-CP.A.3. Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B),* and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A,* and the conditional probability of *B* given *A* is the same as the probability of *B.*			X	X
S-CP.A.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.			X	
S-CP.A.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.			X	X
S-CP.B.6. Find the conditional probability of *A* given *B* as the fraction of B's outcomes that also belong to *A,* and interpret the answer in terms of the model.			X	X
S-CP.B.7. Apply the Addition Rule, P(A or B) = P(A) + P(B) — P(A and B), and interpret the answer in terms of the model.			X	X
S-CP.B.8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(BIA) = P(B)P(AIB), and interpret the answer in terms of the model.				X

	Algebra	Geometry	Algebra II	Pre calculus
S-CP.B.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.				X
Using Probability to Make Decisions (MD)				
S-MD.A.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.				
S-MD.A.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.				
S-MD.A.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.				
S-MD.A.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.				
S-MD.B.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. i. Find the expected payoff for a game of chance. j. Evaluate and compare strategies on the basis of expected values.				
S-MD.B.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).				
S-MD.B.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).				

Appendix A: Curricular Map of High School Student Learning Standards for 21st Century Math Applications Course

	Standard Present in Course	Correlation in Financial Algebra Textbook
NUMBER AND QUANTITY (N)		
The Real Number System (RN)		
N-RN.A.1.Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	**X**	Rational Functions are used in the compound interest and monthly payment formulas in Chapters 2, 3, and 7; pgs. 102-108
N-RN.A.2.Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**X**	Section 2-6
N-RN.B.3.Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**X**	The irrational natural base e is irrational as the limit of exponential function in section 2-6.
Quantities (Q)		
N-Q.A.1.Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	**X**	Sections 3-5, 3-6, 3-7, 8-2, 8-3, 8-6, 9-5, 9-7, 9-8, & 11-1
N-Q.A.2.Define appropriate quantities for the purpose of descriptive modeling.	**X**	Sections 3-5, 8-2, 8-3, 8-4, 9-5, 9-7, 9-8, & 11-1
N-Q.A.3.Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	**X**	Sections 8-2, 9-5, & 9-7
The Complex Number System (CN)		
N-CN.A.1.Know there is a complex number *i* such that $i^2 = —1$, and every complex number has the form *a* + *bi* with *a* and *b* real.		Not Applicable
N-CN.A.2.Use the relation $i^2 = —1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.		Not Applicable

N-CN.A.3.(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
N-CN.B.4.(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.		Not Applicable
N-CN.B.5.(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.		Not Applicable
N-CN.B.6.(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.		Not Applicable
N-CN.C.7.Solve quadratic equations with real coefficients that have complex solutions.		Not Applicable
N-CN.C.8.(+) Extend polynomial identities to the complex numbers.		Not Applicable
N-CN.C.9.(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.		Not Applicable
Vector and Matrix Quantities (VM)		
N-VM.A.1.(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v, 14 114,** v).		Not Applicable
N-VM.A.2.(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.		Not Applicable
N-VM.A.3.(+) Solve problems involving velocity and other quantities that can be represented by vectors.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
N-VM.B.4. (+) Add and subtract vectors. A. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. B. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. C. Understand vector subtraction v— was v + (—w), where —w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.		Not Applicable
N-VM.B.5. (+) Multiply a vector by a scalar. A. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vv) = (mix, cv_i). B. Compute the magnitude of a scalar multiple cv using licv11= iciv. Compute the direction of cv knowing that when Iclv # 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).		Not Applicable
N-VM.C.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.		Section 11-3
N-VM.C.7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.		Not Applicable
N-VM.C.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.		Not Applicable
N-VM.C.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.		Not Applicable
N-VM.C.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
N-VM.C.11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.		Not Applicable
N-VM.C.12. (+) Work with 2 x 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.		Not Applicable
Algebra (A)		
Seeing Structure in Expressions (SSE)		
A-SSE.A.1. Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.	**X**	Used throughout the text when constructing algebraic models for real life situations; Sections 2-1, 2-4, 2-6, 3-1, 3-2, 3-5, 3-6, 3-7, 4-8, 6-3, 6-4, 6-5, 7-3, 7-4, 7-6, 8-9, 9-5, 10-2, 11-1, & 11-4
A-SSE.A.2. Use the structure of an expression to identify ways to rewrite it.	**X**	Section 3-2
A-SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions.	**X**	Topic used in simplifying equations, but not expressions as the standard mandates in sections 2-1, 2-5, 2-6, 2-8, 3-1, 3-2, 4-8, & 8-4,
A-SSE.B.4. Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.		Not Applicable
Arithmetic with Polynomials and Rational Expressions (APR)		
A-APR.A.1.Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.		Not Applicable
A-APR.B.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
A-APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	X	zeros of profit and revenue functions introduced in Chapter 9
A-APR.C.4. Prove polynomial identities and use them to describe numerical relationships.		Not Applicable
A-APR.C.5. (+) Know and apply the Binomial Theorem for the expansion of *(x* + y)" in powers of *x* and y for a positive integer *n,* where *x* and y are any numbers, with coefficients determined for example by Pascal's Triangle.		Not Applicable
A-APR.D.6.Rewrite simple rational expressions in different forms; write $a^{(x)}/_{b(x)}$ in the form $q(x) + {}^{trx)}/\ b(x)$, where a(x), *b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x),* using inspection, long division, or, for the more complicated examples, a computer algebra system.	X	Section 7-3
A-APR.D.7.(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.		Not Applicable
Creating Equations (CED)		
A-CED.A.1.Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	X	Sections 5-4, 8-1, 8-2, 8-6, & 8-8
A-CED.A.2.Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	X	Sections 4-1, 4-5, 4-6, 5-1, 7-1, 8-7, 9-3, 9-4, 9-5, & 9-6
A-CED.A.3.Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	X	Sections 3-1, 4-5, 4-6, 6-1, 6-4,6-5, 7-1, 7-3, 9-4, 9-5, 9-6, 9-6, 9-7, 9-8, & 10-2
A-CED.A.4.Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	X	Sections 2-2, 2-3, 2-8, 4-7, 5-2, & 8-6

	Standard Present in Course	Correlation in Financial Algebra Textbook
Reasoning with Equations and Inequalities (REI)		
A-REI.A.1.Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.		Not Applicable
A-REI.A.2.Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**X**	Sections 4-9 & 9-4
A-REI.B.3.Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**X**	Sections 5-4, 8-2, 8-6, & 8-8
A-REI.B.4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	**X**	Sections 9-5 & 9-6
A-REI.C.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.		Not Applicable
A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	**X**	Section 9-4
A-REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	**X**	Sections 9-6, 9-7, & 9-8
A-REI.C.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.		Not Applicable
A-REI.C.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
A-REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	X	Sections 9-5, 9-6, 9-7, 9-8, &11-3
A-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y= *f(x)* and y = *g(x)* intersect are the solutions of the equation *f(x) = g(x);* find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	X	Sections 9-6, 9-7, & 9-8
A-REI.D.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	X	Section 9-4
FUNCTIONS (F)		
Interpreting Functions (IF)		
F-IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x.* The graph of *f* is the graph of the equation y= *f(x).*	X	Sections 4-1, 8-2, 9-3, & 9-4
F-IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	X	Sections 4-1, 5-1, 5-3, & 6-2
F-IF.A.3.Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.		Not Applicable
F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**	X	Sections 2-8, 4-9, 5-5, 9-3, 10-4, & 11-3
F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it descrbes.*	X	Sections 9-3 & 11-3

	Standard Present in Course	Correlation in Financial Algebra Textbook
F-IF.B.6.Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*	X	Section 4-5
F-IF.C.7.Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step *functions* and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are *available,* and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable *factorizations* are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end *behavior,* and trigonometric functions, showing period, midline, and amplitude.	X	Sections 2-8, 4-1, 4-3, 4-6, 9-3, 9-5, 9-6, 9-7, & 11-3
F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.	X	Chapters 6 & 9; Sections 2-5, 2-7, 2-8, 3-2, 4-6, & 11-2
F-IF.C.9.Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	X	Automobile depreciation functions are interpreted algebraically and graphically in sections 4-5 and 4-6.
Building Functions (BF)		
F-BF.A.1.Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. c. (+) Compose functions.	X	Used throughout the text when constructing algebraic models for real life situations; Chapter 9 makes extensive use of combining function types; Sectons 2-1, 3-1, 5-4, 6-3, 7-3, 7-4, 7-6, 10-2, 11-4
F-BF.A.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	X	Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
F-BF.B.3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.		Not Applicable
F-BF.B.4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.		Not Applicable
F-BF.B.5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving exponents and logarithms.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
Linear, Quadratic and Exponential Models (LE)		
F-LE.A.1.Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal difference over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	X	Used in chapters 3 and 4 when constructing regression models for real life situations; Sections 4-5, 4-6. 5-4, & 7-6
F-LE.A.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).		Not Applicable
F-LE.A.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.		Not Applicable
F-LE.A.4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e;* evaluate the logarithm using technology.		Not Applicable
F-LE.B.5.Interpret the parameters in a linear or exponential function in terms of a context.		Not Applicable
Trigonometric Functions (TF)		
F-TF.A.1.Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.		Not Applicable
F-TF.A.2.Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.		Not Applicable
F-TF.A.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for Tr13, Tr/4 and Tr/6, and use the unit circle to express the values of sine, cosines, and tangent for *x,* IF + *x,* and 2-rr — *x* in terms of their values for *x,* where *x* is anyreal number.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
F-TF.A.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.		Not Applicable
F-TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*		Not Applicable
F-TF.B.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.		Not Applicable
F-TF.B.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*		Not Applicable
F-TF.C.8. Prove the Pythagorean identity $\sin^2(0) + \cos^2(0) = 1$ and use it to find sin(0), cos(e), or tan(0) given sin(0), cos(0), or tan(0) and the quadrant of the angle.		Not Applicable
F-TF.C.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.		Not Applicable
GEOMETRY (G)		
Congruence (CO)		
G.CO.A.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.		Not Applicable
G-CO.A.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).		Not Applicable
G-CO.A.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.		Not Applicable
G-CO.A.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		Not Applicable
G-CO.A.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.		Not Applicable
G-CO.B.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.		Not Applicable
G-CO.B.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
G-CO.B.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.		Not Applicable
G-CO.C.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*		Not Applicable
G-CO.C.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*		Not Applicable
G-CO.C.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*		Not Applicable
G-CO.D.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*		Not Applicable
G-CO.D.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.		
Similarity, Right Triangles, and Trigonometry (SRT)		
G-SRT.A.1. Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.		Not Applicable
G-SRT.A.2.Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.		Not Applicable
G-SRT.A.3.Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.		Not Applicable
G-SRT.B.4.Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
G-SRT.B.5.Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.		Not Applicable
G-SRT.C.6.Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		Not Applicable
G-SRT.C.7.Explain and use the relationship between the sine and cosine of complementary angles.		Not Applicable
G-SRT.C.8.Use trigonometric ratios and Pythagorean Theorem to solve right triangles in applied problems.*		Not Applicable
G-SRT.D.9.(+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.		Not Applicable
G-SRT.D.10.(+) Prove the Laws of Sines and Cosines and use them to solve problems.		Not Applicable
G-SRT.D.11.(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).		Not Applicable
Circles (C)		
G-C.A.1.Prove that all circles are similar.		Not Applicable
G-C.A.2.Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*		Not Applicable
G-C.A.3.Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.		Not Applicable
G-C.A.4.(+) Construct a tangent line from a point outside a given circle to the circle.		Not Applicable
G-C.B.5.Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.		Arc length is introduced in section 4-9 when computing automobile yaw mark lengths. Areas of sectors and regular polygons are examined in section 7-2.
Expressing Geometric Properties with Equations (GPE)		
G-GPE.A.1.Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		Not Applicable
G-GPE.A.2.Derive the equation of a parabola given a focus and directrix.		Not Applicable
G-GPE.A.3.(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
G-GPE.B.4.Use coordinates to prove simple geometric theorems algebraically.		Not Applicable
G-GPE.B.5.Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).		Not Applicable
G-GPE.B.6.Find the point on a directed line segment between two given points that partitions the segment in a given ratio.		Not Applicable
G-GPE.B.7.Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*		Not Applicable
Geometric Measurement and Dimension (GMD)		
G-GMD.A.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*		Not Applicable
G-GMD.A.2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.		Not Applicable
G-GMD.A.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*		Not Applicable
G-GMD.B.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.		Not Applicable
Modeling with Geometry (MG)		
G-MG.A.1.Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*		Not Applicable
G-MG.A.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*		Not Applicable
G-MG.A.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*		Not Applicable
Statistics and Probability (S)		
Interpreting Categorical and Quantitative Data (ID)		
S-ID.A.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
S-ID.A.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	X	Section 4-2
S-ID.A.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	X	Section 4-2
S-ID.A.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	X	Section 4-2
S-ID.B.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.		Not Applicable
S-ID.B.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data (including with the use of technology) to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology c. Fit a linear function for a scatter plot that suggests a linear association.	X	Sections 3-4, 4-6, & 7-1
S-ID.C.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	X	Section 4-5
S-I D.C.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	X	Sections 7-1 & 9-3
S-ID.C.9. Distinguish between correlation and causation.		Not Applicable
Making Inferences and Justifying Conclusions (IC)		
S-IC.A.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
S-IC.A.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.		Not Applicable
S-IC.B.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.		Not Applicable
S-IC.B.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.		Not Applicable
S-IC.B.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.		Not Applicable
S-IC.B.6. Evaluate reports based on data.		Not Applicable
Conditional Probability and the Rules of Probability (CP)		
S-CP.A.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").		Not Applicable
S-CP.A,2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.		Not Applicable
S-CP.A.3. Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B),* and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A,* and the conditional probability of *B* given *A* is the same as the probability of *B.*		Not Applicable
S-CP.A.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.		Not Applicable
S-CP.A.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.		Not Applicable
S-CP.B.6. Find the conditional probability of *A* given *B* as the fraction of B's outcomes that also belong to *A,* and interpret the answer in terms of the model.		Not Applicable
S-CP.B.7. Apply the Addition Rule, P(A or B) = P(A) + P(B) — P(A and B), and interpret the answer in terms of the model.		Not Applicable
S-CP.B.8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(BIA) = P(B)P(AIB), and interpret the answer in terms of the model.		Not Applicable

	Standard Present in Course	Correlation in Financial Algebra Textbook
S-CP.B.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.		Not Applicable
Using Probability to Make Decisions (MD)		
S-MD.A.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	**X**	Section 10-4
S-MD.A.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	**X**	Section 10-4
S-MD.A.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.		Not Applicable
S-MD.A.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.	**X**	Section 10-4
S-MD.B.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. b. Evaluate and compare strategies on the basis of expected values.	**X**	Section 10-4
S-MD.B.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).		Not Applicable
S-MD.B.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).		Not Applicable

Appendix B: Course Syllabus

Cengage Textbook Outline of Units

Unit 1: Income/Banking/Expenses

- Chapter 1: Discretionary Expenses
- Chapter 2: Banking Services
- Chapter 5: Employment Basics
- Chapter 6 Income Taxes

Unit 2: Homeownership /Budget

- Chapter 7: Independent Living
- Chapter 11: Prepare a Budget

Unit 3: Credit/Purchases

- Chapter 3: Consumer Credit
- Chapter 4: Automobile Ownership

Unit 4: Investments/Retirement

- Chapter 8: Stock Market
- Chapter 9: Modeling a Business
- Chapter 10: Planning for Retirement

Appendix C: Webb's Depth-of-Knowledge (DOK) Levels for Mathematics created by Norman Webb

According to Norman L. Webb, Wisconsin Center for Educational Research ("Depth-of-Knowledge Levels for Four Content Areas," March 22, 2002), "interpreting and assigning depth-of-knowledge levels to both objectives within standards and assessment items is an essential requirement of alignment analysis. Four levels of depth-of-knowledge are used for this analysis." Norman Webb's "Depth-of-Knowledge Levels for four Content Areas" include: Language Arts (reading, Writing), Mathematics, Science, and Social Studies.

A general definition for each of the four (Webb) Depth-of-Knowledge levels is followed by Table 1, which provide further specifications and examples for each of the DOK levels. Webb recommends that large scale, on-demand assessments in mathematics should only assess Depth of Knowledge Levels 1, 2, and 3. Depth of Knowledge at Level 4 in mathematics should be reserved for local assessment.

Level 1 (Recall) includes the recall of information such as fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify", "recall", "recognize", "use", and "measure". Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify", "organize", "estimate", "make observations", "collect and display data", and "compare data". These actions imply more than one step. For example, to compare data requires first identifying characteristics of the object or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain", "describe", or "interpret" could be classified at different levels depending on the object of the action. For example, if an item requires students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered

and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level of other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be complex. Students should be required to make several connections-relate ideas *within* the content area or *among* content areas-and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

Math Descriptors- Combined Webb Depth of Knowledge Levels of Mathematics (Webb, 2002), NAEP 2002 Mathematics Levels of Complexity, and Other Descriptors related to NECAP GLEs.

LEVEL 1 RECALL	LEVEL 2 SKILLS/CONCEPTS	LEVEL 3 STRATEGIC THINKING	LEVEL 4 EXTENDED THINKING
Examples represent, but do not constitute all Level 1 mathematics performances: Recall or recognize a fact, definitions, or terms Apply a well-known algorithm Apply a formula Determine the area or perimeter of rectangles or triangles given drawing and labels Identify a plan or three dimensional figure Measure a length Perform a specified or routine procedure Evaluate an expression	**Examples represent, but do not constitute all Level 2 mathematics performances:** Classify plane and three dimensional figures Interpret information from a simple graph Use models to represent mathematical concept Solve a **routine** problem requiring multiple steps, or the application of multiple concepts Compare figures or statements Provide justifications for steps in a solution process Extend a pattern	**Examples represent, but do not constitute all Level 3 mathematics performances:** Interpret information from a complex graph Explain thinking when more than one response is possible Make and/or justify conjectures Develop logical arguments for a concept Use concepts to solve problems Perform procedure with multiple steps and multiple decision points Generalize a pattern Describe, compare and contrast solution methods	**Examples represent, but do not constitute all Level 4 mathematics performances:** Relate mathematical concepts to other content areas Relate mathematical concepts to real-world applications in new situation Apply a mathematical model to illuminate a problem, situation Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results Design a mathematical model to inform and solve a practical or abstract situation ***NOTE:*** **Level 4 requires applying**

Solve a one-step word problem Retrieve information from a table or graph Recall, identify, or make conversions between and among representations or numbers (fractions, decimals, and percent's), or within and between customary and metric measures Locate numbers on a number line, or points on a coordinate grid Solves linear equations Represent math relationships in words, pictures, or symbols	Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps Translate between tables, graphs, words and symbolic notation Select a procedure according to criteria and perform it	Formulate a mathematical model for a complex situation Provide mathematical justifications Solve a multiple step problem, supported with a mathematical explanation that justifies the answer Formulate an original problem, given a situation	**one approach among many to solve problems. Involves complex restructuring of data, establishing and evaluating criteria to solve problems.**